Math 20 Course Pack
Prealgebra
Fall 2014

Instructor: Yolande Petersen

DO NOT BUY THESE NOTES IF YOU HAVE A DIFFERENT INSTRUCTOR

Inside:
- Lecture Notes Outline with writing space for your notes
- Example Test Problems (Petersen style only) for each chapter
- Final Exam Review Problems & Answers

How to assemble these pages in a notebook (3-ring binder needed):
The lecture notes pages are in reverse order, upside down, and punched on the "wrong" side for a reason! My notes read like a book, with the printed side on the left and space for handwritten extra notes on the right. To make your notebook look like mine:
1. If there is a staple, remove it.
2. Separate the "upside down", single-sided section from the "right side up" double-sided section.
3. KEEPING ALL PAGES IN A STACK IN THE SAME ORDER, flip the whole stack of "upside down" pages so that you have the blank back of page 1 on top, holes on the left. Insert these pages into the binder. When you turn the first page, page 1 will be on your left. It should look like a book, with the printed page on the left and the blank page for writing extra notes on the right.
4. Insert the "right-side up" pages into the binder, as you normally would.

Effort was made to minimize the number of pages printed to reduce your cost, while leaving enough space for your notes to be arranged in an orderly way. If you don't like this arrangement, feel free to assemble the pages however you like.

Mrs. Petersen's website: http://peterseny.faculty.mjc.edu
Before you take this class, you may find it helpful to read the document "Teaching Style and Educational Philosophy" to decide whether this instructor is a good match for you. You can find it at the above web address, with the link on the home page under the "What To Expect" heading.
Chapter 1 – Whole Numbers

1.1 Numbers, Notation, and Rounding

Whole Numbers - include the counting (natural numbers) and zero

Digit Names

(skip Objective #2, expanded form)

Number Line – draw a dot on line to represent number

Equations & Inequalities

Equations - have equal sign,
= equal to

Inequalities - have one of the following inequality symbols:
> greater than
< less than
> greater than or equal to
< less than or equal to

Ex

Rounding to a Specified Place Value (Procedure)

1. Look at the next digit after the specified place.
2. If 0 – 4, keep same digit.
   If 5 – 9, round up to next digit (if that digit is 9, the next digit changes also)
3. Keep correct place value/zeroes
Rounding to one non-zero digit - choose the highest place digit
Goal – get a “simple” number which is close to the actual value.

Ex. A car costs Round to one non-zero digit.

### Bar Graphs – good for comparing categories

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedans</td>
<td>38</td>
</tr>
<tr>
<td>SUV’s</td>
<td>24</td>
</tr>
<tr>
<td>Trucks</td>
<td>31</td>
</tr>
</tbody>
</table>

### Line Graphs – good for showing a progression over time

<table>
<thead>
<tr>
<th>Week</th>
<th>Weight in Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>10</td>
<td>146</td>
</tr>
<tr>
<td>20</td>
<td>153</td>
</tr>
<tr>
<td>30</td>
<td>163</td>
</tr>
<tr>
<td>40</td>
<td>180</td>
</tr>
</tbody>
</table>

Try: Make a line graph:

<table>
<thead>
<tr>
<th>Age</th>
<th>Median Income (thousands of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>28</td>
</tr>
<tr>
<td>35</td>
<td>43</td>
</tr>
<tr>
<td>45</td>
<td>56</td>
</tr>
<tr>
<td>55</td>
<td>62</td>
</tr>
<tr>
<td>65</td>
<td>58</td>
</tr>
</tbody>
</table>

### Pie Graphs – good for showing parts of a whole

<table>
<thead>
<tr>
<th>Category</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1.2 Adding & Subtracting Whole Numbers; Solving Equations

Properties of Addition
1. **Commutative** - you can change order without changing value

2. **Associative** – you can change grouping without changing value

3. **Identity** – element that "does nothing" – does not change result
   The adding identity is

4. **Inverse** – adding and subtracting are inverses

Estimating Sums & Differences - (book's method)
1. Round the smallest number to one non-zero digit
2. Round all other numbers to same place value

Ex

Another method (for place values that differ > 1 place): Use highest 2 place values

Ex  What can I buy at the mall with $100 (tax-free day)?

Try: Estimate the cost of buying a house $126,995

<table>
<thead>
<tr>
<th>Item</th>
<th>Actual Cost</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outfit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shoes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earrings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shampoo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gumball</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item</th>
<th>Actual Cost</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>$126,995</td>
<td></td>
</tr>
<tr>
<td>Repairs</td>
<td>21,550</td>
<td></td>
</tr>
<tr>
<td>Closing</td>
<td>2,169</td>
<td></td>
</tr>
<tr>
<td>Landscape</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>New lock</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Applications (word problems) for Addition
Key words: add, sum, plus, total, increased by, more than, in all, all together

Perimeter – the total distance around the outside of a shape, found by adding

Applications for Subtraction – the inverse of addition
Key words: subtract, difference, minus, remove, take away, less than, decreased,

Solving for missing (unknown amounts)
251 + ? = 326 We solve this using the inverse property:

Rule: when you move a term from one side of an equation to the other, change the sign or operation.

Try:

Applications
Ex
1.3 Multiplying Whole Numbers; Exponents
(If you haven’t memorized mult. tables, you should)
Multiplication: shortcut for repeated addition

Properties of Multiplication
1. Commutative - you can change order without changing value

2. Associative – you can change grouping without changing value

3. Identity – element that "does nothing" – does not change result
The multiplying identity is

4. Inverse – multiplying and are inverses

5. Distributive
   \[ a(b + c) = ab + ac \]
   \[ (a + b)c = ac + bc \]
   Similar to doubling a recipe:

Estimating Products (different from sums)
1. Round each factor to one non-zero digit (highest place value)
2. Multiply

Applications for Multiplication
Key words: multiply, product, times, of, by, each
**Rectangular Array** – an arrangement of rows and columns.

**Combinations** – The total number of combinations is the product of the possible choices for each category.

**Exponential Form** - repeated multiplications

\[ 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = \]

**Solving Applications Given a Formula**
1. Substitute variables with known values
2. Solve (calculate)
1.4 Dividing Whole Numbers, Solving
Division – inverse of multiplication

We use the inverse property in solving:

Division is also a shortcut for repeated subtraction

When a number is divided evenly by another number, it is divisible by that number.

Division properties
1. \( n \div 1 = \frac{n}{1} = n \)
2. \( n \div 0 = \frac{n}{0} \) undefined
3. \( 0 \div n = \frac{0}{n} = 0 \)
4. \( n \div n = \frac{n}{n} = 1 \)
5. \( 0 \div 0 = \frac{0}{0} \) indeterminate (sometimes undefined, sometimes real)
**Divisibility Rules**

- A number is divisible by 2 if the number is

- A number is divisible by 3 if the digits add to a number divisible by 3

- A number is divisible by 4 if the last 2 digits form a number divisible by 4

- A number is divisible by 5 if the number ends in

**Applications for Division**

Key words: divide, quotient, distribute, evenly split, into, per, over
1.5 Order of Operations; Mean, Median & Mode

The order you use when performing operations may affect the outcome

Standard Order of Operations (PEMDAS)
1. Parentheses/Grouping Symbols (including horizontal division bars, brackets, braces)
2. Exponents
3. Multiplication & Division (equal priority, in order, from left to right)
4. Addition & Subtraction (equal priority, in order, from left to right)

Note: When no operator is shown between quantities, the implied operation is:
**Mean (Average) – Procedure for calculating**
1. Find the sum of values
2. Divide by how many values

**Median – for an odd number of values**
1. Rewrite the list of values in order (smallest to largest or largest to smallest)
2. Choose the middle value (the median)

**Median – a more sophisticated approach for an even number of values**
1. Rewrite the list in order
2. Look at the middle 2 values.
3. Calculate the average of the middle 2 values.

**Mode**
1. The most common value is the mode. If there are 2 (or more) most common values, there are 2 (or more) modes. If no value is more common than any other, there is no mode.
1.6 More With Formulas

Perimeter is a _________________. Can you measure it with a ruler?

Area is a _________________. Can you cover it with square tiles?

Volume is a _________________. Can you fill it with cubes?

Perimeter of a Rectangle:

Area of a Rectangle:

Area of a Parallelogram:

Volume of a Box:

Are each of the following quantities measured in length, area, or volume?

1. carpet 
2. baseboard 
3. gas in a car 
4. distance to LA 
5. amount of fencing 
6. house size 
7. refrigerator size 
8. grass/sod 
9. compost
Composite Shapes

Perimeter:

Another way: Observer that the “cut-out sides” are the same length as the “outside” sides

Area:

Another way:
### 2.1 Introduction to Integers (positive & negative whole numbers)

Natural (counting numbers):

Whole numbers:

Integers:

<table>
<thead>
<tr>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
</table>

#### Number Line

![Number Line](image)

#### Graphs

![Graph of Profit (millions)](image)

#### Inequalities

**Ex a** Write the correct relationship for each pair:

#### Absolute Value

Absolute value - the distance from a number to 0 on the number line
2.2 Adding Integers
Like signs - add amounts, keep same sign

Different signs (2 integers) – subtract amounts, choose sign of larger amount

Adding More Than 2 Numbers - 2 methods
Method 1: Add pairs from left to right, one at a time.

Method 2: Add all positives, then all negatives separately. Combine the positive and negative totals

Additive Inverse (also called opposite) – has same amount (absolute value), opposite sign
Applications

Ex  A book shelf is supported by 2 blocks. The forces exerted are as shown.

Add the forces. What does the result mean?

Ex  Trey’s bank account begins with $348. He withdraws $160, deposits $457, then withdraws $300.
1) Write an expression using signed numbers to represent these transactions.

2) What is his final balance?
2.3 **Subtracting Integers** - same as adding the opposite of a number

**Subtracting Procedure**
1. Change the subtraction sign to addition
2. Change the sign of the number subtracted
3. Add as before

Ex

Observe: Subtracting a negative has the same result as adding a positive

**Solving Equations**
Recall: When you move a term to the other side, reverse the sign or operation (inverse)

Applications

**Financial/Business:** \( N = R - C \)
- \( C \) = cost (money spent)
- \( R \) = revenue (money received)
- \( N \) = net

Profit occurs when the net is

Loss occurs when the net is

Ex A start-up business spends $7000 to purchase supplies and $1000/month in rent and utilities. The first year, $40,000 in merchandise is sold. What is the net profit or loss?
2.4 Multiplying & Dividing Integers; Exponents, Square Roots, Solving

Recall: When no operator is shown _________________ is implied.

Rules for Multiplying Integers:

**same sign:**

**opposite signs:**

Ex

Why is negative x positive = negative?

Why is negative x negative = positive?

More than 2 negative signs multiplied or divided:

even number of negative signs:

odd number of negative signs:

**Evaluating Exponential Form**

\((-5)^2 =\)

\(-5^2 =\)

\((-2)^3 =\)

\(-2^3 =\)
Rules for Dividing Integers:

same sign:

opposite signs:

Solving – use inverse operation (related equation)

Square Root - the number which, squared, gives the final number (also, the length of a square with a given area)

\(( x )^2 = 25\)

Every positive number has 2 square roots

Words: square roots of 81

Symbols: \(\sqrt{81}\)

When a square root is written as a symbol, it has only one root, positive or negative, but not both.

Ex
Square root of a negative number:
\[ \sqrt{-36} \]

Some perfect squares (helpful to memorize)

<table>
<thead>
<tr>
<th>Root number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
</tr>
</tbody>
</table>

Applications

Ex. A car loan is taken out, and the amount to be paid back (including interest) is $36,000. If the term of the loan is 6 years, how much is repaid each month. Write amounts as signed numbers.
2.5 Order of Operations with Integers (including negative numbers)

- PEMDAS rules still apply. Absolute value bars and square root symbols are considered ____________________, which should be done __________.

- Be care of (–) signs, especially with exponents.

Recall: \((-4)^2 = \)
\(- 4^2 = \)
2.6 Additional Applications & Problem Solving

Net/Revenue/Cost
Formula:

Voltage/Current/Resistance
Formula: \( V = i R \), where
- \( V \) = voltage (in volts)
- \( i \) = current (in amperes or amps)
- \( R \) = resistance (in ohms or )

Distance/Rate/Time
Formula: \( d = r t \)
3.1 Translating & Evaluating Expressions

### Equation
- Has equal sign
- Process:
- Examples:

### Expression
- No equal sign
- Processes:
- Examples:

**Translating Phrases to Expressions**

We represent real quantities as variables, because these quantities keep changing.

4 important words:
- sum
- difference
- product
- quotient

**Examples (more in book)**

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>the sum of x and 2</td>
<td>the difference of x and 2</td>
</tr>
<tr>
<td>x plus y</td>
<td>x minus y</td>
</tr>
<tr>
<td>5 more than y</td>
<td>5 less than y</td>
</tr>
<tr>
<td>3 added to some number</td>
<td>3 subtracted from some number</td>
</tr>
<tr>
<td>10 increased by a number</td>
<td>10 decreased by a number</td>
</tr>
</tbody>
</table>

**Note:** Addition is commutative; subtraction is not.

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>the product of 2 numbers</td>
<td>the quotient of 2 numbers</td>
</tr>
<tr>
<td>x times 3 ⇔ x multiplied by 3</td>
<td>x divided by 3</td>
</tr>
<tr>
<td>twice some number</td>
<td>x divided into 3</td>
</tr>
<tr>
<td>triple a number</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Multiplication is commutative; division is not.
### Exponents

- $x$ squared ⇔ the square of $x$
- A number cubed ⇔ the cube of a number
- $y$ to the $4^{th}$ power
- $p$ raised to the $7^{th}$ power

### Roots

- The square root of a number
- The cube root of a number

---

**Evaluating Expressions** - find a number value

1. Replace each variable with a number. If it helps, you can replace each variable with parentheses first, then insert the number into the parentheses.
2. Calculate the final number using the correct order of operations.

Note: “Find the value of the expression” has the same meaning as “Evaluate”
3.2 Introduction to Monomials: Combining Like Terms

Monomial (term) – a number, variable, or product of numbers and variables (which may be raised to whole-number exponents). They contain NO _______________________.

Examples of monomials:

Examples of things that are NOT monomials:

A monomial has 2 parts:
1. ______________________ - the number part, including the sign
2. ______________________ - the letters, including exponents

Ex a Find the coefficient and variable part of each term below:

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Variable Part</th>
<th>Degree of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: A term always has _______________________. It does not always have _______________________.

Degree of a term - the sum of the exponents of the variables for a single term

Polynomials with One Variable – most common
Polynomial – has 1 or more terms added or subtracted.
Examples of polynomials:

Binomial - has
Trinomial – has
Descending Order - A polynomial written with highest powers first.

Degree of a polynomial – the degree of the highest power term only (there may be several terms)

Like terms - terms that have exactly the same variable parts (including exponents)
Ex Are the following pairs of terms like or unlike?
3.3 Adding and Subtracting Polynomials

Simplest Form – an expression with the fewest number of terms and symbols
- get rid of parentheses
- combine all like terms

Polynomials in 1 variable (compare terms of polynomial to digits in arithmetic)

Adding Polynomials
Horizontal Method - Gather like terms in descending order, then combine

Vertical Method - Stack like terms in descending order, then combine

Perimeter - add the lengths of the sides

Subtracting Polynomials - Change to an addition problem by
1. Changing subtract to add
2. Changing the sign of ALL terms in 2nd parentheses (subtracted terms)
3.4 Exponent Rules; Multiplying Polynomials

Product Rule of Exponents

\[ n^a \cdot n^b = n^{a+b} \]

Ex a

Power to a Power Rule of Exponents

\[ (n^a)^b = n^{ab} \]

Ex b

Multiplying Monomials
1. Gather numbers, then like variables together
2. Multiply coefficients, then each type of variable separately.

Raising a Monomial to a Power
1. Raise the coefficient to the power and evaluate
2. Raise each variable to the power and simplify

Multiplying a Polynomial by a Monomial
1. Use the distributive law to multiply each term inside the parentheses
2. Simplify as above.
**Multiplying 2 Polynomials: Book's Method**
1. Multiply first term in first polynomial by second polynomial
2. Multiply each of the other terms by second polynomial
3. Gather like terms

**Multiplying 2 Polynomials: Box Method**
1. Make a box, with each polynomial on one side
2. Multiply terms, filling each box
3. Gather like terms

**Multiplying 2 Binomials – special case of 2 polynomials**
FOIL - First, Outer, Inner, Last

**Ex**

**Conjugates** - 2 binomials that have the same 2 terms, but the second terms have opposite signs.

**Rule:** the product of conjugates is the difference of squares
3.5 Prime Numbers and Greatest Common Factors (GCF’s)

Prime Number – a natural number greater than 1 whose only factors are 1 and itself. (a number that can’t be “broken down” to smaller factors)

Note: All even numbers, except 2, are NOT prime, since ________ is a factor.

Composite Number – a natural number that has factors other than 1 or itself. Every composite number CAN be “broken down” to smaller factors, and eventually to prime factors.

Finding ALL the factors of a number
Ex Find all the factors of 60.

How do we know when to stop?

Deciding if a number is prime or composite - test each factor to see if there are any smaller factors

Finding the Prime Factorization of a Number
1. Break down the number into its smallest (prime) factors
2. Write the factors as a product, using exponents if needed
Finding the Greatest Common Factor by Lists
1. List all factors of each number
2. Find the largest factor common to both lists

Finding the GCF by factoring
1. Write each number in its prime factored form, including exponents
2. Choose common bases only; choose the lowest power common to both numbers
3. Multiply
3.6 Exponents Rules; Dividing Polynomials/Introduction to Factoring

Quotient Rule of Exponents - When dividing like bases, subtract exponents

Zero Rule of Exponents – any number raised to the zero power =

Dividing Monomials
1. Group coefficients and similar variables together
2. Divide using exponent rules (subtract exponents) or cancellation

Dividing Polynomials by a Monomial
1. Separate terms to divide each term by the monomial
2. Simplify as before

Finding an Unknown Factor
Long way: Divide the product by the known factor
Short way: “Eyeball” the product to see what’s missing
3.7 Additional Applications and Problem Solving

Guidelines for Word Problems
1. Decide which formula is appropriate. Write it.
2. Substitute the known numbers or quantities into the variables of the formula.
3. Solve for the unknown variable.

Perimeter, Area, Volume formulas

P = sum of all sides (any shape)
P = 2L + 2W (rectangle)
A = LW  (rectangle)
A = bh  (parallelogram)
V = LWH

Ex a  Given the shape with dimensions shown, find

\[
\begin{array}{c}
\text{x - 3} \\
\text{2x + 1}
\end{array}
\]

Surface Area
SA = 2LW + 2WH + 2HL

Falling Objects
\[ h = -16t^2 + h_0 \]  \( h = \) current height, \( t = \) time, \( h_0 = \) original height
Net/Cost/Revenue - Formula:

Ex: Amy buys jewelry-making tools for $100. Materials cost $10 for earrings, $15 for bracelets, and $20 for pendants. She charges $20 for earrings, $30 for bracelets, and $35 for pendants.

1) Write an equation describing cost, where E = # of sets of earrings, B = # of bracelets, and P = # pendants sold.

2) Write an equation describing the revenue

3) Write an equation describing the net

4) What is the net if she sells 5 sets of earrings, 1 bracelet and 3 pendants? Is it a profit or loss?

5) Suppose she fills an order next week for 3 sets of earrings, 4 bracelets, and 2 pendants. If she can reuse the same tools (reducing costs), what is her net?
### 4.1 Equations and Solutions
Recall the contrast between equations and expressions:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has</td>
<td>No</td>
</tr>
<tr>
<td>Process:</td>
<td>Process(es):</td>
</tr>
<tr>
<td>Looks like:</td>
<td>Looks like</td>
</tr>
</tbody>
</table>

**Solutions of an Equation**

- **Solve** - get a number that makes the equation true
- **Solution** - a number that makes the equation true

Checking to see if a number is a solution - Procedure
1. Replace the variable(s) with the number
2. Simplify and determine if the equation is true
4.2 The Addition/Subtraction Property of Equality
Linear Equation – an equation where each variable term has degree 1
linear:

non-linear:

Addition/Subtraction Property of Equality - you can add or subtract the same amount from both sides without changing the solution
A = B is equivalent to A – C = B – C
or A + C = B + C

Goal in Solving: Get the variable by itself on one side of equation (isolate the variable); get the number on the other side

Solving Procedure for equations with 1 complication
1. Combine like terms on the left side of the = sign; then combine like terms on the right
2. Look at the variable. Decide what is “junk”.
3. Decide how the “junk” is connected to the variable. Do the reverse to “undo” the junk.
4. Isolate the variable
5. Check

Solving with variables on 2 sides of equation – get rid of smaller variable term first
4.3 The Multiplication/Division Property of Equality

Multiplication/Division Property of Equality - you can multiply or divide by the same amount on both sides without changing the solution

A = B is equivalent to \( AC = BC \)

or \( \frac{A}{C} = \frac{B}{C} \)

“Undo” junk by doing the reverse operation

Solving with both added/subtracted & multiplied/divided junk

Solving Procedure (summary)
1. Simplify if needed (clear parentheses, combine like terms)
2. Get all variable terms on one side, by adding or subtracting from both sides; combine into one variable term
3. Get rid of added/subtracted “junk”
4. Get rid of multiplied/divided “junk”
4.4 Translating Word Sentences to Equations
Recall 4 important words: sum, difference, product, quotient

Recap Examples
Addition
the sum of a number and 2
5 more than a number
3 added to t
a number increased by 10

Subtraction
the difference of 3 and a number
3 less than y
7 subtracted from a number
x decreased by 8

Multiplication
the product of 4 and a number
6 times p
half of x
twice a number

Division
the quotient of a number and 11
4 divided by a number
$15 per 5 gallons

Equal
4.5 Applications and Problem Solving
Procedure for solving
1. Let one quantity be an expression “built around” x, and the other be x.
2. Assign quantities to the expression.
3. Make an equation using the expressions
4. Solve and ANSWER THE QUESTION
5. Check (optional)

Triangle and Angle Definitions
1. Equilateral triangle – triangle that has
2. Isosceles triangle – triangle that has
3. Supplementary angles – angles that add up to
4. Complementary angles – angles that add up to
5. Congruent angles – angles with

Solving for 1 type using a formula
Money Formula: value \cdot number = amount
Common constructions (algebraic expressions) for 2 types
1. “one side is 4 ft more than the other side”

2. “there are twice as many children as adults”

3. “there are 10 coins total”

Solving for 2 types using a table

<table>
<thead>
<tr>
<th>category</th>
<th>value</th>
<th>number</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Ex**

<table>
<thead>
<tr>
<th>category</th>
<th>value</th>
<th>number</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.1 Fractions, Mixed Numbers, Rational Expressions

Fraction – part out of a whole

\[
\frac{\text{part}}{\text{whole}} = \frac{\text{numerator}}{\text{denominator}}
\]

Examples of fractions:

- **Proper fraction** – fraction where the numerator is less than the denominator

- **Improper fraction** - fraction where the numerator is greater than or equal to the denominator.

- **Mixed number** – an integer combined with a (proper) fraction

Which is better?

In general,

- improper fractions are better for
- mixed numbers are better for

Converting Improper Fractions to Mixed Numbers

1. Divide numerator by denominator using long division
2. Write quotient as whole number part, write remainder over the denominator for the proper fraction part.
Converting Mixed Numbers to Improper Fractions
1. Multiply the whole number part by the denominator
2. Add the numerator to the product
3. Put the sum over the denominator

Rational number – a number that can be written as a/b
Fraction – a number that actually is written as a/b

Rational numbers include:
- all
- all
- all

Graphing a fraction on a number line

Equivalent Fractions – Reducing and Building Up
reducing - cancel common factors

building up – multiply by the missing factor

Comparing Fractions using > < or =
To compare fractions it’s helpful to have the same denominators
5.2 Simplifying Fractions and Rational Expressions

lowest terms – a fraction whose numerator and denominator have no common factors

equivalent fractions – fractions that have the same value when reduced to lowest terms

Reducing to Lowest Terms (2 methods)
Method 1 – Divide numerator and denominator by common factors (or the GCF) until no common factors remain

Method 2 – Write prime factors and cancel

Reducing Improper Fractions
1. Cancel common factors (required)
2. Convert to a mixed number (optional)
Acceptable Reduced Forms:
Acceptable  Not Acceptable

Not all answers are required to be shown in mixed number form.

Convention: If the problem begins with improper fractions, give the final answer as an improper fraction (same is true of mixed numbers), unless the final answer is asked for in a different form.

Reducing Expressions with Variables

Caution: Cannot cancel added terms (do not confuse addition with multiplied factors, which can be canceled).

\[
\frac{5x^2}{5x^2 + 2} \quad \frac{5x^2}{5x^2 \cdot 2}
\]

Compare to:
5.3 Multiplying Fractions, Mixed Numbers, Rational Expressions

Sign of a Fraction - A fraction is considered a single number and should have only one sign and only one opposite; for example the opposite of $\frac{3}{5}$ is $\frac{-3}{5}$, not $\frac{-3}{5}$.

Acceptable form for proper and improper fractions:

Acceptable form for mixed numbers:

Translating Expressions & Equations - “Of” usually means multiply

Multiplying Procedure
1. Reduce by cancelling any numerator with any denominator until all common factors are gone.
2. Multiply 2 numerators and put in the numerator
3. Multiply 2 denominators and put in the denominator

Multiplying Mixed Numbers
1. Convert to an improper fraction
2. Cancel common factors
3. Multiply
4. Convert back to a mixed number for final answer (what you started with)
Estimating Products (operation is ____________) – book says round to nearest half, but I’ll accept nearest whole number, unless the rounded number is 0

1. Round each mixed number to the nearest whole number.
   - In the fraction part, look at denominator number – what is half the denominator?
   - If the numerator is smaller than half the denominator, the fraction is < ½ (round down)
   - If the numerator greater than or equal to half the denom., the fraction is ≥ ½ (round up)

2. If there is no integer part and the fraction is less than ½, keep the fraction as is, since factors in products should not be rounded to 0.

3. Multiply the numbers

**Multiplying Rational Expressions (polynomials)** - cancel variables as well as numbers

**Fractions Raised to a Power** – raise each factor in numerator & denominator separately

**Applications**

**Area of a Triangle**: \( A = \frac{1}{2}bh \)
Ex 2/3 of all students in the Central Valley graduate from high school. If 1/6 of HS graduates earn a BA/BS degree, what fraction of all CV students earn BA/BS degrees?

Ex If ¾ of the stores in a mall are open for business, and there are 300 stores:
1) How many stores are open?
2) How many are closed?
3) What fraction are closed?
4) If 1/3 of the closed stores are being remodeled, what fraction of all stores are being remodeled?
5) How many stores are being remodeled?
5.4 Dividing Fractions, Mixed Numbers, Rational Expressions

reciprocals – 2 numbers whose product is 1; the “flip” or “upside down” of a fraction

Dividing Fractions – Procedure
1. Keep first fraction the same. Invert the second fraction and change division to multiplication (invert and multiply).
2. Cancel any common factors
3. Multiply numerators and denominators
4. Simplify

Dividing Mixed Numbers
1. Convert the mixed numbers to improper fractions
2. Divide as above
3. Convert back to a mixed number

Complex fractions (more than 2 layers) - can be written as a division problem
Dividing Rational Expressions (variables)

Square Roots of Fractions – Procedure
1. Simplify fraction if possible
2. Separate numerator and denominator
3. Find the square roots separately

Solving Fractional Equations (multiplied/divided “junk” only)
Method 1: Divide both sides by the fraction

Method 2: Multiply both sides by the reciprocal

Applications
**5.5 Least Common Multiples**

**multiple** – the product of a whole number and the original number (a multiple’s size is the same as or ________________ than the original number)

Multiples of 6: 6, 12,

Multiples of 8:

Common multiples:

**Least common multiple** – smallest multiple that is common to both (or all) numbers

**Finding the LCM by listing**
1. Write lists of the multiples of each number
2. Compare the lists until the first common multiple is found

**LCM Shortcuts**
1. If there are all numbers have no common factors, the LCM is the product
2. If one number is a perfect multiple of all the numbers, the “big” number is the LCM.

**Finding the LCM by Prime Factorization**
1. Find the prime factorization of every number
2. Choose each unique base
3. Write largest exponent for each base
4. Multiply the numbers to get the LCM

**Finding the Greatest Common Factor**
1. Find the prime factorization
2. Choose only bases common to all the numbers
3. Write smallest exponent for each base
4. Multiply the numbers to get the GCF

**Factor** - a “broken down” piece, same as or smaller than the number

**Multiple** – a multiplied product, same as or larger than the number
Sometimes we want to “force” fractions to have the same denominator

Building an equivalent fraction to an LCD – Procedure
1. Find the LCD. Write a new fraction with the LCD and an empty (unknown) numerator
2. Find the missing factor needed to change the old denominator into the new denominator. If you can’t “eyeball” it, divide the new denominator by the old.
3. Multiply both numerator and denominator by the missing factor to make the new fractions
## 5.6 Adding and Subtracting Fractions, Mixed Numbers, and Rational Expressions

Two or more fractions must have the same denominator to be added (e.g. same size of pizza slices)

**Adding/Subtracting with Same Denominator** (Procedure)
1. Add or subtract numerators – write in numerator
2. Keep same denominator
3. Reduce if needed

Caution:

**Adding/Subtracting with Different Denominators** (Procedure)
1. Find the LCD
2. Build all fractions to the LCD
3. Add/subtract numerators, keep same denominator
4. Simplify

Reminder: LCDs are only needed for adding/subtracting fractions. Multiplying fractions does not require having the same denominators.

**Adding/Subtracting Mixed Numbers**

**Method 1:** Convert to improper fractions, combine, then convert back to mixed numbers
Method 2: Combine whole number and fraction parts separately. Borrow or carry if needed

**Estimating Sums** – (operation is ) – rounding to zero IS acceptable
1. Round to the nearest whole number - up if fraction , down if fraction
2. Combine numbers

**Solving fractional equations** (added/subtracted “junk” only) - get rid of “junk” by doing the reverse process
5.7 Order of Operations - Evaluating & Simplifying Fractions & Rat. Expressions

Recall order of operations: PEMDAS

For multiplying – improper fractions are best
For adding – either mixed numbers or improper fractions can be used

Ex a

Evaluate – substitute a number for each variable

Area of a Trapezoid

\[ A = \frac{1}{2} h(a+b) \] or \[ A = \frac{1}{2}(a+b)h \]

Note: \( \frac{1}{2}(a+b) \) is the average of a and b

Adding/Subtracting Polynomials – gather like terms together, then add/subtract
5.8 Solving Equations – both multiplied/divided AND added/subtracted “junk”
Solving Procedure – different from simplifying expressions
1. Find LCD
2. Multiply both sides by LCD to GET RID of fractions (do NOT build up to keep fractions)
3. Solve as before

Translating sentences to equations
“Of”

Applications
Ex  Ray’s scores are 87, 52, and 76. What must he score on the next test for a “C” average?
6.1 Decimals and Rational Numbers

Converting decimals to fractions or mixed numbers
1. Write the numbers to the left of the point as a whole number
2. Write the digits to the right as numerator
3. Write the place value in the denominator
4. Reduce if needed

Trick: you can tack extra zeroes onto the end if needed

Word Names – decimal words are similar to the fraction they represent
1. Number left of decimal point:
2. Point:
3. Number right of point:

Graphing on a number line
Ex

Ex
Comparing Decimals using $>$ or $<$
1. If needed, tack on zeroes to make the decimals equal length
2. Compare digits beginning immediately after the point (if needed, tack on zeroes to make numbers the same length)
3. When digits differ, the larger number gives the larger amount

Rounding
1. Look at the specified digit, then look at the next place value (immediately after the specified one)
2. If the next digit is
   0 – 4, keep the desired digit
   5 – 9, round up

Ex Round to the nearest
a) thousandth
b) hundredth
c) whole number (unit)
d) ten
e) hundred

Ex Round to the nearest
a) hundredth
b) tenth
c) whole number (unit)
d) ten
e) hundred
6.2 Adding and Subtracting Decimals
Adding Procedure
1. Stack numbers so place values align (line up decimal points)
2. Add the numbers keeping decimal point in the same place
   (If it helps, tack on zeroes to make the same number of places in each decimal)

Subtracting Procedure
1. Stack the numbers, with greater abs. value (amount) on top.
2. Subtract digits, tacking on zeroes if necessary
3. Determine the final sign of the number

Adding/Subtracting Polynomials
1. Put like terms together
2. Combine numbers for each type
Solving (added/subtracted “junk” only) – Isolate the variable

Application

Ex Find the final balance

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Credits</th>
<th>Debits</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$ 11.36</td>
</tr>
<tr>
<td>Pay check</td>
<td>1605.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PG &amp; E</td>
<td></td>
<td>189.21</td>
<td></td>
</tr>
<tr>
<td>Rent</td>
<td></td>
<td>825.00</td>
<td></td>
</tr>
<tr>
<td>Costco</td>
<td></td>
<td>211.52</td>
<td></td>
</tr>
<tr>
<td>Refund</td>
<td></td>
<td>20.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: There is no transaction in the line before the first balance.
6.3 Multiplying Decimals: Exponents
Multiplying decimals gives the same value as multiplying equivalent fractions

Multiplying Procedure
1. Stack the 2 numbers, one on top of the other. You don’t need to line up decimal points.
2. Multiply the numbers as if they were whole numbers.
3. Count the number of places to the right of all decimals.
4. Move the decimal point to the left that number of places
5. Fill in zeroes if necessary

Multiplying/Dividing Shortcuts – powers of 10
Multiplying by a power of 10 makes a number _________________ (move the decimal point to the right the same # of places as the power).

Dividing by a power of 10 makes a number _________________ (move the decimal point to the left the same # of places as the power).
Scientific Notation – a special form of writing numbers. It is most useful for very large (or very small) numbers.

Form:
- Exactly one non-zero digit left of decimal point
- Zero or more digits right of decimal point
- Multiplied by power of 10 to express place value

Ex Which of the following are in scientific notation?

Converting Scientific Notation to Place Value Form
1. Decide if exponent will make the decimal larger or smaller (pos. or neg. exponent)
2. Move decimal point the number of places in exponent
3. Fill in zeroes if necessary

Converting Place Value Form to Scientific Notation
1. Put decimal point after the first non-zero digit
2. Count how many places you moved
3. Write that number as the exponent (positive exp. for numbers $\geq 1$)

Polynomials

Unit Price - Price per single unit (1 in denominator)
6.4 Dividing Decimals; Square Roots
Dividing decimals by a whole number
1. Divide as if the numerator were a whole number
2. Write the quotient above, lining up decimal points
3. Continue dividing until:
   a. There is no remainder OR
   b. The decimal repeats itself OR
   c. If you are told to round, you have divided one place past the place you are rounding to.

Dividing decimals by a decimal
1. Write as a fraction
2. Move decimal point in both numerator & denom. until the denom. is a whole number.
3. Divide as above

Converting fractions and mixed numbers to decimals
1. If mixed, write integer part to the left of the decimal point
2. Divide out the fraction by long division and write it to the right of the point
Finding Square Roots of Non-Perfect Squares

irrational number – can’t be expressed as a/b. The decimal never repeats, never ends
The best way to find a square root:
  1. approximately
  2. exactly

Decimal Perfect Squares and Roots -- Observe the squares of some decimals:

$(.2)^2 = .04 \quad 1\text{ place }\rightarrow 2\text{ places}$

$(.02)^2 = .0004$

$(.002)^2 = .000004$

Conclusion: A perfect square has a(n) ______________ number of decimal places
A square root has ___________ the number of places as the square.

Polynomials

Applications

Circles

circumference – length of the outside of a circle (perimeter, curved).
radius – distance from center to edge of a circle
diameter – distance across a circle from edge to edge, through the center
Formula: $C = 2\pi r$ or $C = \pi d$
6.5 Order of Operations with Decimals

Recall: PEMDAS

Mixed Decimals and Fractions – change to all decimals or to all fractions

Note: Changing to all fractions is often easier

- If fractions change to a repeating decimal, must use fractions, e.g. \( \frac{1}{3} \)
- If fractions are not easy to convert (too much long division), fractions are better
  e.g. \( \frac{5}{32} \)
- Changing to decimals is only better if the fractions are “easy to see” as decimals

Weighted Mean - sometimes, certain categories count more “heavily” than others

GPA (grade point average) A = 4.0, B = 3.0, C = 2.0, D = 1.0

How heavily a class is weighted depends on the number of units (credits).
GPA = total grade points/# of units

Ex Find the GPA of the following schedule to the nearest tenth.

<table>
<thead>
<tr>
<th>Class</th>
<th>Units</th>
<th>Grade</th>
<th>Grade value</th>
<th>Total grade points</th>
</tr>
</thead>
<tbody>
<tr>
<td>English 101</td>
<td>3</td>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math 20</td>
<td>5</td>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biology 111</td>
<td>4</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Music 121</td>
<td>1</td>
<td>A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Ex  In my class, exams are weighted 50%, homework is weighted 25%, and the final is weighted 25%. If Pat’s exam avg. is 80, the final exam is 75, and homework is 40, what is her final average and grade?

Area formulas
Triangle: \( A = \frac{1}{2}bh \)

Trapezoid: \( A = \frac{1}{2}h(a + b) \)

Circle: \( A = \pi r^2 \)

Volume of a cylinder/straight-sided object
\[ V = Ah, \quad \text{where} \quad A = \text{area of a “slice”} \]
\[ h = \text{height} \]
6.6 Solving with Decimals

2 Methods for isolating the variable

Method 1: Get rid of all decimals by moving the point the same number of places in EVERY term.

Method 2: Keep all decimals until the end

Translating to algebra

Pythagorean Theorem

- legs – 2 sides touching the right angle
- hypotenuse – side opposite the right angle
Applications
Ex A phone company charges $40/month with 500 free minutes. After 500 minutes, $0.50/minute is charged. If Ray's bill is $152.50, how many minutes were used?

Using a table (2 types)

Recall algebraic expressions for common constructions:
“5 more hours than last week”
“twice as many hot dogs as burgers”
“12 items all together”

Ex Ana has 20 quarters and dimes totaling $3.35. How many of each coin does she have?

<table>
<thead>
<tr>
<th>category</th>
<th>value</th>
<th>number</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Try Candy costs $0.75 and soda costs $1.25. If $19.50 is spent and twice as many sodas as candy bars are bought, how many of each is bought?
Estimation Supplement
There are 2 goals of estimation:
1. To round complicated numbers so they are simple enough to do calculations in your head
2. To keep close to the original number so you have a “ballpark” idea of the value.

Based on these 2 conditions, what would be a good estimate of $4982.19?

Estimating sums differs from estimating products, especially when one number is relatively large compared to the other number(s).

Estimating Sums

Compare the sum: 10 + 0.1 with the number 10.

```
   10
 + 0.1
```

\[
10 + 0.1 = 10.1
\]

Adding 0.1 to 10 hardly changes the 10, so we can ignore the 0.1 when adding. When rounding, we should keep the highest 1 or 2 place values, since adding more than 2 digits becomes complicated.

Estimating Sums Procedure
1. Choose the largest number and round it to the highest 1 or 2 place values.
2. Round all other numbers to those same 1 or 2 place values. Some small numbers may become zero.
3. Add

Ex a  Estimate the sum:
House: $123,995.00
Repairs $ 21,326.22
Landscaping $ 1,242.55
Air freshener $ 2.39
Estimating Products
Now compare the number 10 with the product 10 x 0.1.

\[ 10 \ (= 10 \times 1) \]

\[ 10 \times 0.1 \]

But multiplying 10 by 0.1 makes it much smaller than 10 (big change). So we must keep both numbers in the calculation when doing multiplication. We should round each number to one non-zero digit, because multiplying with 2 or more digits becomes complicated.

Estimating Products Procedure
1. Round each number to one non-zero digit, keeping correct place value
2. Multiply

\textbf{Ex b} Estimate the product: (689.62983)(21.14380)(0.03482)

\textbf{Ex c} Estimate the cost of 980 doses of medication if each dose is 0.00617 mg and the cost is $49.85/mg. (Actual cost is $301.42)

Estimating Quotients Procedure
1. Round the divisor (denominator) to one non-zero digit, keeping correct place value.
2. Round the numerator to a number easily divided by the denominator.

\textbf{Ex d} Estimate the quotient \[ \frac{19,042}{321} \]

Note: You can quickly check the answers to any complicated problem for “ballpark” values using estimates.
7.1 Ratios and Rates

**Ratio** – a comparison of 2 quantities using a fraction (quotient)
The ratio of a to b: \( \frac{a}{b} \) or a:b

Ratios may use **like** units (cancel units) or **unlike** units (keep units)

**Rate** - a ratio comparing 2 different measurements (?)

**Unit Ratio** – ratio where the denominator number is 1
Ex. A college has 5000 students and 250 faculty. Find the student to faculty ratio.

Ex. May owes $30,000 on a house, and value of the house (asset) is $120,000.
Find the debt to asset ratio (this ratio is sometimes called the “debt ratio”).

**Unit Rate** = denominator number is 1

**Unit Price** – cost per unit (cost/unit)

**Probability**
1. Write the number of desired choices in numerator
2. Write the number of all possible choices in denominator
7.2 Proportions

Proportion – 2 ratios that equal each other: \( \frac{a}{b} = \frac{c}{d} \)

Cross Multiplying (cross products)

If \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \)

Testing whether ratios are proportions (cross multiply)

Solving Proportions
1. Cross multiply to eliminate fractions
2. Solve as before

Cancelling Shortcuts
1. You can cancel straight up/down (this is the same as reducing a fraction)
2. You can cancel straight across (same as multiplying both sides of an equation)
3. You can’t cancel diagonally – can’t “cross cancel”

Solving proportion problems chart

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Quantity 1</th>
<th>Quantity 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.3 American Measurement (English Units)

<table>
<thead>
<tr>
<th>ENGLISH MEASURES AND EQUIVALENTS</th>
<th>Length</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 inches (in) = 1 foot (ft)</td>
<td></td>
<td>60 seconds (sec) = 1 minute (min)</td>
</tr>
<tr>
<td>3 feet (ft) = 1 yard (yd)</td>
<td></td>
<td>60 minutes (min) = 1 hour (hr)</td>
</tr>
<tr>
<td>5280 feet (ft) = 1 miles</td>
<td></td>
<td>24 hours (hr) = 1 day</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7 days = 1 week</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liquid Volume</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 teaspoons (tsp) = 1 tablespoon (Tbsp)</td>
<td>16 ounces (oz) = 1 pound (lb)</td>
</tr>
<tr>
<td>8 fluid ounces (fl. oz. or just oz) = 1 cup (c)</td>
<td>2000 pounds (lb) = 1 ton</td>
</tr>
<tr>
<td>2 cups (c) = 1 pint (pt)</td>
<td></td>
</tr>
<tr>
<td>2 pints (pt) = 1 quart (qt)</td>
<td></td>
</tr>
<tr>
<td>4 quarts (qt) = 1 gallon</td>
<td></td>
</tr>
</tbody>
</table>

dimensional analysis – converting from one type of unit to another using unit fractions
unit fractions - fractions that equal one, formed by dividing 2 sides of an equation

Conversion Procedure
1. Write equations containing the original and desired quantities
2. Write the original quantity on top
3. Multiply by unit fractions, arranging units to cancel
4. Cancel units, put numbers together
7.4 Metric Measurement

Metric system – more systematic than American/English units (easier calculations)

<table>
<thead>
<tr>
<th>METRIC LENGTH</th>
<th>VOLUME</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo</td>
<td>1 kilometer (km) = 1000 m</td>
<td>1 kiloliter (kl) = 1000 l</td>
</tr>
<tr>
<td>hecto</td>
<td>1 hectometer (hm) = 100 m</td>
<td>1 hectoliter (hl) = 100 l</td>
</tr>
<tr>
<td>deka</td>
<td>1 dekameter (dam) = 10 m</td>
<td>1 dekaliter (dal) = 10 l</td>
</tr>
<tr>
<td>base</td>
<td>1 meter = 1 meter</td>
<td>1 liter = 1 liter</td>
</tr>
<tr>
<td>deci</td>
<td>10 decimeter (dm) = 1 m</td>
<td>10 deciliter (dl) = 1 l</td>
</tr>
<tr>
<td>centi</td>
<td>100 centimeter (cm) = 1 m</td>
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Other Common Prefixes

kilobytes (KB)          milliseconds (msed)
megabytes (MB)          microseconds (µsèd)
gigabytes (GB)          nanoseconds (nsec)
terabytes (TB)          picoseconds (psec)

Miscellaneous metric equivalencies

1 metric ton = 1000 kg
1 milliliter (ml) = 1 cubic centimeter (cc)
### 7.5 Converting between American (English) and Metric Systems

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**Temperature formula**: \( F = \frac{9}{5}C + 32 \), where \( F \) = Fahrenheit temp., \( C \) = Celsius temp.
7.6 Applications (skip mortgage ratios)

Sometimes we make our own conversion equations that are not on standardized charts, based on the information given.

Ex  A veterinarian administers Butorphanol (a sedative) to a 1000 lb. horse. The recommended dosage is 0.02 mg/kg. A vial of the drug has a concentration of 10 mg/ml. How much of the drug should be given?

Ex  A certain cereal contains 6 grams of fiber per serving. The recommended intake of fiber is 24 grams per day, there are 12 servings of cereal in the box, and each box costs $6. How much does it cost for a year’s worth of fiber using this cereal?
8.1 Introduction to Percent
percent – out of 100
57% =

Converting percent to a fraction
1. Replace % with /100
2. Move decimal point if need (top & bottom)
3. Reduce if needed

Converting percent to a decimal
1. Move decimal point 2 places to the left (smaller)
2. Remove % symbol

Converting a fraction or decimal to percent
1. First, write as a decimal (if a repeating decimal, write out at least 3 places)
2. Multiply by 100% (move decimal point 2 places and tack on % symbol)
   (Note: 100% = → no change)
8.2 Translating Percent Sentences

of $\rightarrow$ multiply
out of $\rightarrow$ divide
is $\rightarrow$ equals

Method 1: Use Multiplying Equation ("of")

percent of whole is part  $\quad$ OR  $\quad$ part is percent of whole

<table>
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<tr>
<th>Example type</th>
<th>Word phrase</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>unknown part</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unknown whole</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unknown percent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: When solving or performing calculations, represent percent as a decimal

Method 2: Use Dividing Equation ("out of")

percent = $\frac{\text{part}}{\text{whole}}$, and percent = $\frac{\text{percent number}}{100}$
8.3 Solving Percent Problems
We will make use of the relationships:

1) part = percent of whole and 2) percent = \[
\frac{\text{part}}{\text{whole}} = \frac{\text{percent number}}{100}
\]

Finding Part
Ex A restaurant tip is 15% of the bill. If $60 is spent on food, what is the tip?

Finding Whole
Ex A college accepts 40% of all applicants. If 500 students were accepted, how many applied?

Finding Percent
Ex 2000 DVD’s are produced, and 37 are damaged. What percent can be sold (undamaged)?
8.4 Solving Problems – Percent Increase or Decrease
Note: Both increase and decrease are a “part” of the original amount

**Increase**
\[
\text{increase} = \text{percent} \cdot \text{old} \\
\text{new} = \text{old} + \text{increase} =
\]

**words to show increase** – increased by, up, raise(d), add, bonus, over, above, markup

**Decrease**
\[
\text{decrease} = \text{percent} \cdot \text{old} \\
\text{new} = \text{old} - \text{decrease} =
\]

**words to show decrease** - discount, sale, off, down, reduce (reduction), deduction, loss, pay cut
8.5 Solving Problems with Interest

Principal, P – amount invested
Interest, I – amount added to principal
Interest rate, r – percent used to calculate interest – usually annual
Simple interest – interest calculated for a year using principal, rate, and time (same amount of interest is paid each year)
Balance, B (most books call this Amount, A) – total of principal and interest

Simple Interest Formula:  \[ I = Prt \]

Balance Formula

\[ B = P + I = \]
9.2 The Rectangular Coordinate System

axis – number line used to locate a point
point – a location in space
ordered pair (x,y) – the coordinates of a point (x is always first)

Ex a  Find the coordinates of the points shown

Ex b  Plot the points

Quadrants - 4 regions defined by the x and y axes
Exc Which quadrant is each point in?

(3, -2)

(-10, -20)

(4, 0)

(0, -7)

(-52, 37)

**Midpoint** – the point halfway between 2 points on a line segment

For each coordinate, find the average of the 2 values:
x-coordinate average:

y-coordinate average:
9.3 Graphing Linear Equations (2 variables)

Linear Equation - each variable term has degree 1
- $x^2 - 4x + 7 = 0$
- $x + 8 = 12$
- $x - y = 5$

A solution is a number that gives a true equation.
- For $x + 8 = 12$, the solution looks like:
- For $x - y = 5$, a solution must have

Finding a Solution (Procedure)
An equation with 2 variables may have many solutions. To find one:
1. Choose any number for either $x$ or $y$.
2. Plug that value into the equation to calculate the number for the other variable. Write as an ordered pair.

The solutions of a linear equation can be graphed as points. It takes at least _____ points to determine a line

Graphing a Linear Equation (procedure)
1. Find 2 (or more) solutions to the equation
2. Plot the solutions as points
3. Draw the line through the points
x and y intercepts
x-intercept: point where the line crosses the x-axis (y = 0)
y-intercept: point where the line crosses the y-axis (x = 0)
To find the intercepts:
1. Set opposite coordinate = 0
2. Solve for the desired coordinate
3. Write ordered pair with both coordinates

3 exceptions
1. Vertical Line
2. Horizontal Line
3. Line through origin
**Slope Supplement**

A linear equation can be written several ways including:

1. General Form: $Ax + By = C$, for example $3x + y = 2$
2. Slope-Intercept Form: $y = mx + b$, for example $y = -3x + 2$

Note that the equations $3x + y = 2$ and $y = -3x + 2$ are different forms of the same equation.

For $y = mx + b$,

- $m =$ slope
- $b =$ y-intercept (y coordinate only)

**What is slope?**

Generically speaking, it's the slant or steepness of an object, like a hill, roof, or line. In the linear equation above, the symbol "m" is used to represent slope for a line, and the value depends on the direction and amount of slant.

- If $m$ is positive, the line ________ from left to right
- If $m$ is negative, the line ________ from left to right
- If $m$ has a large absolute value, the line rises or falls __________
- If $m$ has a small absolute value, the line rises or falls __________

**Ex a**

We can also get a general sense of the location of a line by looking at "b"

- If $b$ is positive, the line intersects the y-axis in the positive region, above the x-axis
- If $b$ is negative, the line intersects the y-axis in the negative region, below the x-axis

**Ex b**
Slope as a ratio

Slope is sometimes defined as \( \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} \).

Other words which mean the same thing as slope:
- grade (of a road)
- pitch (of a roof)

Ex c What percent grade does a hill have if it rises 300 feet over a distance of 5000 feet?

Ex d If the pitch of a roof is 25%, what is the distance from the horizontal support to the peak if the horizontal support spans 80 feet all the way across?

Slope formula

\[ m = \frac{y_2 - y_1}{x_2 - x_1}, \] where \((x_1, y_1)\) and \((x_2, y_2)\) are the coordinates of a point.

Ex e Find the slope of the line through the points \((4, -1)\) and \((-3, 0)\), and graph the line.
Ex f  Find the slope of the line through the points (-3, 1) and (2,1), and graph the line.

Ex g  Find the slope of the line through the points (-2, 7) and (-2, 4), and graph the line.

Finding Slope from an Equation
Procedure:
   1. Isolate y on the left. The equation now looks like $y = mx + b$.
   2. The coefficient of x is the slope

Ex h  Find the slope of $y = -\frac{1}{2}x + 5$

Ex i  Find the slope of $2x - 3y = 6$
Slope Supplement Exercises (do all)

For # 1 - 6, tell whether a line with the given slope will be rising, falling, horizontal, or vertical.
1. \( m = -3 \)
2. \( m = 0 \)
3. \( m = -\frac{2}{3} \)
4. \( m = \frac{1}{4} \)
5. \( m = \text{undefined} \)
6. \( m = 7 \)

For each of the following lines (\( y = mx + b \)) tell whether a) \( m \) is positive or negative and b) whether \( b \) is positive or negative.

7. \[ \text{Diagram} \]
8. \[ \text{Diagram} \]
9. \[ \text{Diagram} \]
10. \[ \text{Diagram} \]

11. If a road has an 8% grade up a hill, and the horizontal run is 2000 feet from the edge of the hill to the center, how high is the peak?
12. If skater's ramp has the following dimensions, find the slope in percent:

```
     3 feet
  ___________
    |       |
  8 feet
    |   __ |
```

13. If a roof has the following dimensions, find the slope in percent:

```
     5 ft
  ___________
    |       |
 80 ft
    |   __ |
```

14. Find the slope of the line through the points (-7, 4) and (2, 4).
15. Find the slope of the line through the points (0, 6) and (2, 3).
16. Find the slope of the line through the points (1, -3) and (1, -1).
17. Find the slope of the line through the points (5, 1) and (3, -3).
18. Find the slope of the line \( y = -5x + 7 \)
19. Find the slope of the line \( 3x - y = 4 \)
20. Find the slope of the line \( 2x + 5y = 9 \)
## Conversion Charts for Exams

### English Measures and Equivalents

<table>
<thead>
<tr>
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<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 inches (in) = 1 foot (ft)</td>
<td>60 seconds (sec) = 1 minute (min)</td>
</tr>
<tr>
<td>3 feet (ft) = 1 yard (yd)</td>
<td>60 minutes (min) = 1 hour (hr)</td>
</tr>
<tr>
<td>5280 feet (ft) = 1 miles</td>
<td>24 hours (hr) = 1 day</td>
</tr>
<tr>
<td></td>
<td>7 days = 1 week</td>
</tr>
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### Liquid Volume

<table>
<thead>
<tr>
<th>Liquid Volume</th>
<th>Weight</th>
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<tr>
<td>3 teaspoons (tsp) = 1 tablespoon (Tbsp)</td>
<td>16 ounces (oz) = 1 pound (lb)</td>
</tr>
<tr>
<td>8 fluid ounces (fl. oz. or just oz) = 1 cup (c)</td>
<td>2000 pounds (lb) = 1 ton</td>
</tr>
<tr>
<td>2 cups (c) = 1 pint (pt)</td>
<td></td>
</tr>
<tr>
<td>2 pints (pt) = 1 quart (qt)</td>
<td></td>
</tr>
<tr>
<td>4 quarts (qt) = 1 gallon</td>
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### Metric Length, Volume, and Weight

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<tr>
<td>deka 1 dekameter (dam) = 10 m</td>
<td>1 dekaliter (dal) = 10 ℓ</td>
<td>1 dekagram (dag) = 10 g</td>
</tr>
<tr>
<td>base 1 meter = 1 meter</td>
<td>1 liter = 1 ℓ</td>
<td>1 gram = 1 gram</td>
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<tr>
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### Misc. Formulas

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1 milliliter (ml) = 1 cubic centimeter (cc)

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