Math 90 Hybrid Course Notes

Summer 2015
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How to use these notes

The notes and example problems cover all content that I would normally cover in face-to-face (f2f) course. If you can do every example problem, you should have reasonable coverage of the course. My personal recommendation:

1. Look over the notes before each class. Do all the examples you know how to do easily, and make note of the ones that are difficult for you
2. In class, take notes on the problems that are presented. If time permits, students may request to see a few additional difficult problems.
3. After lecture, try to complete all the problems in the notes, using videos if necessary.
4. If you still have trouble, post your question in the discussion forum, where a classmate or the instructor can answer it.
5. Check your notes with the posted completed notes.
6. Steps 1-5 are equivalent to what you .would learn from previewing and attending class every day in a f2f course. Now you should be ready to complete the homework.

Explanation of symbols:

1. Asterisks (*) indicate problems that I intend to cover in class. Some problems without an asterisk will also be covered, if time permits. If time is tight, I may skip a * problem.
2. When an example has 2 problems with “OR”, the first problem is what I would prefer to present in class, and the second is a similar problem with a Khan Academy video . If time allows, I will use the first example in class; otherwise the second example with video can be used to learn the concept.
3. Problems with a corresponding Khan Academy video available are highlighted in blue.
4. Problems with a corresponding Petersen video will be highlighted in yellow (will be used in later sections).

R. 1 Introduction to Algebraic Expressions

Topics include:

- Sets of numbers
    - Natural #'s: {1, 2, 3, ….}
    - Whole #'s: {0, 1, 2, 3, ….}
    - Integers: {… - 3, - 2, - 1, 0, 1, 2, 3, ….}
    - Rational #'s: {n| n = a/b, where a and b are integers, b $\neq$ 0}
- Order of Operations
  1. Parentheses/Grouping Symbols (including radicals, absolute value, etc.)
  2. Exponents
  3. Multiply & Divide
  4. Add & Subtract

  Example a: Simplify $11+\frac{|3-19|}{3^2-1}-7$

- Laws/Properties:
    - Commutative
        - Addition: a + b = b + a
        - Multiplication: ab = ba
    - Associative
        - Addition: (a + b) + c = a + (b + c)
        - Multiplication: (ab)c = a(bc)
    - Distributive: a(b + c) = ab + ac
    - Identity
        - Addition: 0 + a = a → additive identity is ______
        - Multiplication: 1 x a = a → multiplicative identity is ______
    - Inverse
        - Addition: a + (-a) = 0 → additive inverse is _____
        - Multiplication: a x (1/a) = 1 → multiplicative inverse is _____
    - Multiplication by 0: 0 x a =
    - Multiplication by -1: -1 x a = -1(a) =
- Equivalent fractions → both reduced forms will generally be accepted on tests

  $\frac{100}{6}=16\frac{4}{6}=\frac{50}{3}=16\frac{2}{3}$
- Combining like terms;

  Ex b Simplify: $(4x^2 + x - 5) - 2(x^2 - 7x - 3)$

R. 2 Equations, Inequalities, and Problem-Solving

Solving Equations - Isolate the Variable

Use the Add. & Mult. Prop. of Equality to "undo" addition & multiplication

Ex a Solve: 4(1 – 3x) = 9 – 7x OR 5x – 11 = 42

3 possible outcomes:

1. One solution (conditional) Solve: 3x + 1 = 7 → x = 2 (one solution)

2. No solution (contradiction) Solve x + 3 = x → 3 = 0 (no solution)

3. Infinitely many sol. (identity/dependent system) Solve x = x → 0 = 0 (inf. sol.)

*Solving Inequalities

Algebraic	Set Builder Notation	Interval Notation
• $x > 3$		
• $x \le -2$		
• $80 \le x < 90$		

Goal in solving: Isolate x

Caution 1: If you multiply or divide by a negative number, the inequality symbol __________

Caution 2: Subtracting is not the same as multiplying by a negative #.

Ex b Solve and graph on a number line, giving your solution in set builder AND interval notation: $2y - (4y - 3) \ge 10$ OR $\frac{2}{3} > -4y - 8\frac{1}{3}$

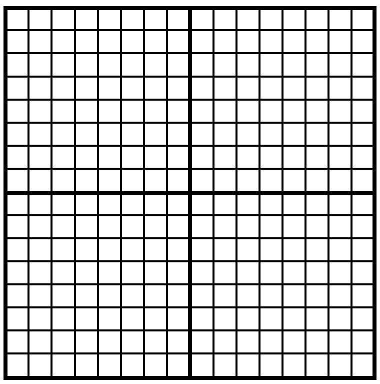
R.3 Introduction to Graphing

3 Forms of Linear Equations

1. General Form: Ax + By = C, e.g. 3x + 2y = 12
2. Slope-Intercept: y = mx + b, e.g., y = -3/2x + 6 (m & b useful landmarks)
3. Point- Slope : $y - y_1 = m(x - x_1)$ (not a final answer form), e.g (y - 3) = -3/2(x - 2)

x-intercept: point where the line crosses the x-axis (y = 0)
y-intercept: point where the line crosses the y-axis (x = 0)

Ex a **Graph the equation: 9x + 16y = 72 by calculating and plotting intercepts**



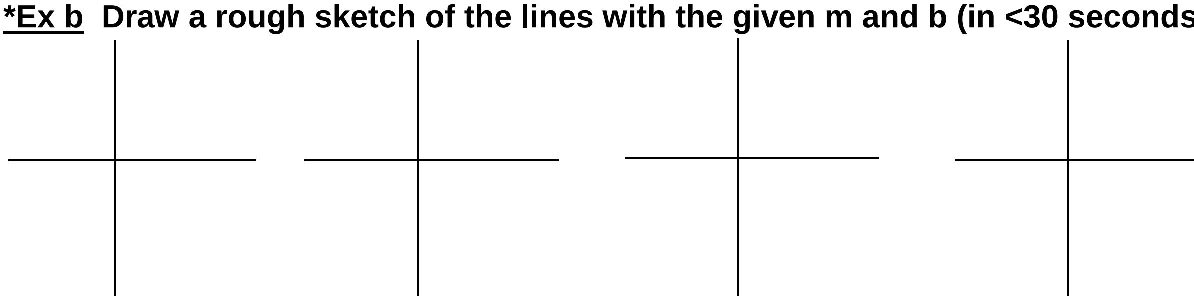
Slope = m = $\frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$

m $= \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are the coordinates of a point.

Generic slope concepts:
Positive slope: line __________; Negative slope: line ___________

Steep lines have slopes with ________________, "gradual" lines have
____

***Ex b Draw a rough sketch of the lines with the given m and b (in <30 seconds):**

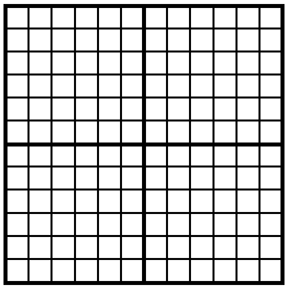


m = 3, b = 0 m = 1/2, b = -3 m = -3, b = 4 m = -1/2, b = -1

Graphing a Line using Slope-Intercept Form (anchor and count)

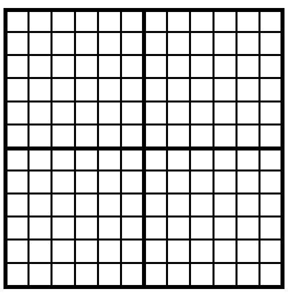
1. Use b to anchor a point on the y axis
2. Use m to determine how many vertical and horizontal units to count for another point.

Ex c **Graph the equation:** $y = -\frac{2}{3}x + 3$ **OR** $y = \frac{1}{3}x - 2$

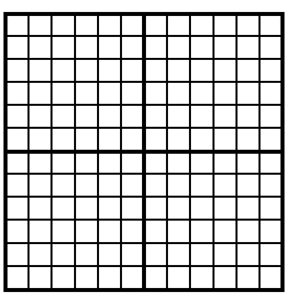


Vertical and Horizontal Lines

Ex d **Find the intercepts, slope, and graph of the equation y = - 3**



Ex e **Find the intercepts, slope, and graph of the equation x = 2**



Finding the Equation of a Line
When you see "Find the equation of the line that…" use Point-Slope Form:
Point – Slope Form – useful tool for intermediate work, but not for final answer
$(y - y_1) = m(x - x_1)$, where m = slope
(x_1, y_1) = coordinates of a point
x & y remain as variables

*<u>**Ex f**</u> **Find the equation of the line thru (-1, -4) and (3,2) in slope-intercept form OR Find the equation of the line with m = 2, passing through (-7, 5) in slope-int. form**

To get an answer in General Form (Ax + By = C):
1. Get rid of fractions
2. Get x and y terms on one side, number on other side
3. Get x positive

<u>**Ex g**</u> **Convert the point-slope equation above to general form.**

<u>Parallel and Perpendicular Lines</u>
(Given 2 lines with slopes m_1 and m_2)
1. The lines are parallel if their slopes are
2. The lines are perpendicular if their slopes:
   - have opposite sign
   - are reciprocals

To find slope: 1) Isolate y; 2) From y = mx + b, use m, the coefficient of x

<u>***Ex h**</u> **Determine whether the lines below are parallel, perpendicular, or neither:**

y = 2x – 5
2x – 4y = 11

R.4 Polynomials and Factoring

Exponent Rules (integer exponents)

1. Product: $b^m \cdot b^n = b^{m+n}$
2. Power to a power: $(b^m)^n = b^{mn}$
3. Product to a power: $(ab)^n = a^n b^n$
4. Quotient to a power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. Quotient: $\frac{b^n}{b^m} = b^{n-m}$
6. Zero Exponent: $b^o = 1$
7. Negative Exponent: $b^{-n} = \frac{1}{b^n}$

Ex a Simplify: $\left(\frac{2x^{-2}y^{-3}}{x^5}\right)^{-2}$ **OR** $(x^{-3})(5^{-2})(x^2)$

Polynomial Vocabulary

monomial – a number, variable, or product of numbers and variables (which may be raised to whole-number exponents). We often use the word "term" to have the same meaning as "monomial" (although there are exceptions).

polynomial – 1 or more terms added or subtracted

coefficient – the number part of a term

degree of a **term** - for terms with one variable, the exponent of the variable; for terms with 2 or more variables (rare), the sum of the exponents on the variables

descending order - a polynomial written with highest powers first.

leading term - the highest power term; the first term of a descending order polynomial

degree of a **polynomial** – the degree of the highest power term only

like terms - terms that have exactly the same variable parts (including exponents)

Ex b Write the polynomial in descending order, fill in the chart for each term and give the degree of the polynomial: $x^2 - x + 3x^4 + 6$

Term	Coefficient	Variable Part	Degree of Term	Degree of Polynomial

Add/Subtract Polynomials - combine like terms

Ex c Simplify: $(x^3 + 3x - 6) + (-2x^2 + x - 2) - (3x - 4)$

Multiply Polynomials

Ex d $(5a - 2)(4a^2 + 3a - 1)$

- FOIL - works for ________________________________
  Ex e $(3x + 2)(4x - 7)$

- Special Products (should memorize)
  - $(A + B)(A - B) =$
  - $(A + B)^2 =$
  - $(A - B)^2 =$

Ex f $(6x - 5y)^2 =$

Division by a Monomial - separate terms and cancel factors in each term

Ex g Simplify: $\dfrac{18x^4 - 3x^2 + 6x - 4}{6x}$

Procedure- Dividing by Polynomials

1. Divide the 2 leading terms. Put quotient piece above bar.
2. Multiply by the divisor
3. Subtract
4. Repeat until terms are used up
5. Write remainder over divisor

Ex h Divide $\frac{6x^3 - x^2 + 5}{2x + 1} =$ $\quad 2x + 1 \overline{)\, 6x^3 - x^2 \qquad\qquad + 5}$

Synthetic Division - a shortcut (Textbook: Appendix C, page 989)

Procedure:

1. Write coefficients of polynomial inside (upside down division bar)
2. Write the divisor number in front, taking the opposite sign
3. Bring down the first coefficient
4. Multiply the first coefficient by the divisor number, add to next coefficient
5. Repeat until all numbers are used
6. Rewrite the polynomial using the numbers as coefficients, reducing the highest power by 1 degree.

Ex i Divide $\frac{3x^3 + 4x^2 - 2x - 1}{x + 4}$

Conditions for synthetic division:

1. Divisor must be a binomial (2 terms only)
2. The binomial is linear (no exponents), with coefficient of x = 1

   Divisors for problems using synthetic division look like:

   Divisors for problems that can't use synthetic division look like:

An example that CANNOT use synthetic division (why?) – use traditional long division

Ex j (long division): $x^2 - x + 1 \overline{)\, x^3 + \qquad 5x - 4}$

R.5 Factoring

Factoring: Greatest Common Factors (GCF) – reverse distributive law
Goal: Take out the largest multiplier possible from every term

1. Find GCF (usually found by examining the numbers, then variables in each term). Write the GCF at the front of the expression.
2. Divide the GCF out of each term, and write the "leftovers" in parentheses

Ex a Factor $10cd^2 + 25c^3d^2$

Factoring trinomials
A trinomial with x^2 as leading term factors to 2 binomials:

$x^2 \pm bx \pm c = (x \pm \quad)(x \pm \quad)$

Our job: Find the 2 numbers in the binomials (including signs)

1. The 2 binomials will be 2 factors of c (write all factor pairs of c if necessary)
2. The 2 factors will have a sum or difference of b
   - If c is positive, 2 factors have same sign and b is a sum. Both signs are positive if b is positive, both signs are neg. if b is neg.
   - If c is negative, 2 factors have different signs and b is a difference

Ex b Factor: $x^2 - 10x + 9$, $x^2 - 11x + 24$, $x^2 + 5x - 14$, $-x^2 + 18x - 72$

Factoring by grouping - most common for 4 terms or trinomials with ax^2 as leading term where "a" can't be factored out:

Ex c Factor $x^3 - 4x^2 - 3x + 12$

Ex d Factor: $4x^2 + 25x - 21$

Factoring: Difference of Squares & Sum/Difference of Cubes (should memorize)

1. $A^2 - B^2 =$

2. $A^3 + B^3 =$

3. $A^3 - B^3 =$

Cautions: $A^2 - B^2 \neq (A - B)^2$
$A^3 + B^3 \neq (A + B)^3$
$A^3 - B^3 \neq (A - B)^3$

Ex e Factor $40c^3 - 5d^3$

Solving – set each factor = 0

Ex f $s^2 - 2s - 35 = 0$

Ex g $4x^2 = 20x - 25$

R.6 Rational Expressions and Equations

Rational Expression – $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$ (polynomial fraction)
Zeroes in the denominator are to be avoided. (Try $5 \div 0$ on your calculator)
Zeroes in the denominator are sometimes called

***Ex a Find the values where $\frac{x-4}{x^3-7x^2+10x}$ is undefined**

Reducing/Cancelling Common Factors
Common factors (multiplied) can be cancelled
Common terms (added or subtracted) cannot be cancelled

Ex b Simplify: $\frac{x^2-9}{5x+15}$ and state the domain. (Contrast with) $\frac{3x^2+7x+5}{3x^2+2x-10}$

Canceling opposite factors:
Ex c Simplify: $\frac{x-5}{5-x}$ and state the domain.

Multiplying/Dividing – For division, 1) keep 1st fraction same, 2) change division to mult., 3) flip second fraction (keep, change, flip). For both operations, factor, cancel common factors, and gather leftover factors.

Ex d Simplify $\frac{x^2+7x+10}{3x+6} \div \frac{x^2+2x-15}{6x-6}$ OR $\frac{2p+6}{p+5} \div \frac{10}{4p+20}$

Adding & Subtracting Rational Expressions

Case 1: (easy) If same denominators, keep denominator, add numerators

Case 2: If different denominators:

1. Factor
2. Find the LCD by writing each prime factor to the highest power (remember that the LCD is a Least Common Multiple, and is typically BIGGER than each factor)
3. Write new fractions with same denominator by building each to the LCD

Ex e **Simplify:** $\frac{5}{6x^4}+\frac{7}{3x^2}$

***Ex f** **Simplify:** $\frac{1}{x+5}-\frac{2}{2x-10}+\frac{2x}{x^2-25}$

Complex Fractions - Method 1

1. Get 4 clearly separated layers to get 2 simple, separate fractions.
2. Convert division to multiplication

***Ex g** **Simplify** $\dfrac{\frac{x-5}{5}}{\frac{1}{x}-\frac{1}{5}}$

Solving Fractional Equations
Goal: Use LCD to multiply both sides and get rid of fractions
Caution: Check equation for bad points

Ex h **Solve:** $\frac{x+2}{5}-\frac{x-1}{6}=\frac{3}{5}$

***Ex i** **Solve:** $\frac{x}{x-3}+\frac{x+3}{x+3}=\frac{18}{x^2-9}$

Proportions – use cross multiplication: If $\frac{a}{b}=\frac{c}{d}$, then ad = bc

Ex j **Solve:** $\frac{5}{x-1}=\frac{3}{x+3}$

Applications
1. Distance = rate X time
2. Work = rate X time, but "5 hours to paint a room" is often interpreted as
   r = 1/5 room/hour

***Ex k** **Ian takes 5 hours to rake and bag leaves, and Kyandre takes 3 hours. Working together, how long does it take to complete the job?**

<u>*Ex l</u> Two hoses are used to fill a fish pond. Together they take 12 minutes to fill the pond. Alone, one hose takes 10 minutes longer than the other. How long does it take each one to fill the pond alone?

<u>*Ex m</u> My husband drives 10 mph faster than me and travels 420 miles in the same time it takes me to drive 360 miles. How fast is each of us going?