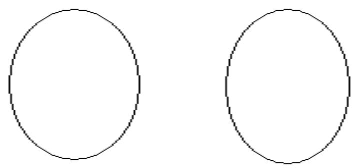
7.1 Introduction to Functions

A correspondence connects 2 sets of quantities (usually x and y) to each other.

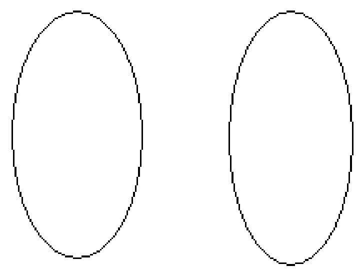


Domain – set of all possible x values (inputs)
Range – set of all possible y values (outputs)

Some examples of correspondences

1. A set of ordered pairs

Age	Weight (lb)
4	42
7	61
9	75
12	92
9	68



2. A vending machine

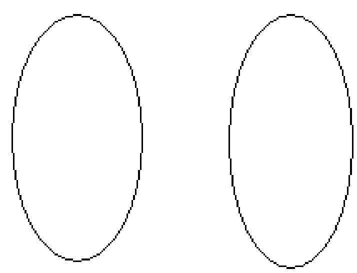
A →

B →

C →

D →

E →



Function – a correspondence where each input (x) has exactly one output (y)

- never 2 or more outputs for same input
- OK to have 2 inputs produce same output
- a function is predictable

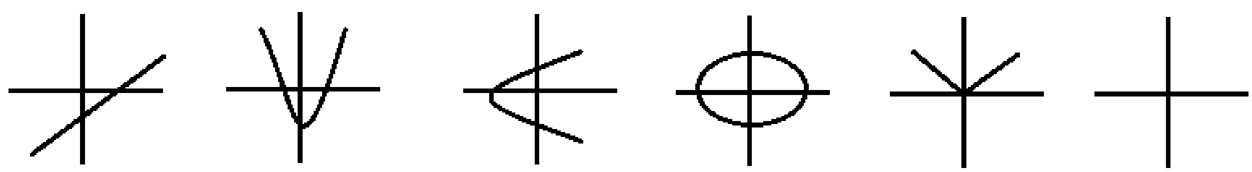
Ordered Pairs: Deciding if ordered pairs are functions

1. Check to see if 2 or more pairs have same inputs. (If never same inputs, no conflict, so it's a function)
2. If same inputs but different outputs, not a function.

<u>Ex a</u> Does the set of pairs {(90,4), (72,2), (94,4), (61,1)} define a function?

<u>Ex b</u> Does the set of pairs {(90,4), (72,2), (90,3)} define a function?

<u>Vertical Line Test</u> - A graph is not a function if any vertical line cuts the graph at more than one point
<u>Ex c</u> Which of the following graphs are functions?



All 2-dimensional inequalities are not functions

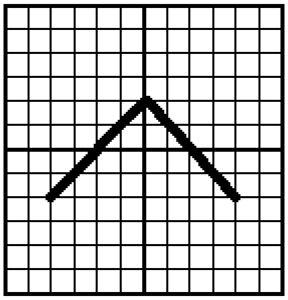
<u>Function Notation and Equations</u>
f – name of function
x – domain
f(x) -- range of function
We often replace y with f(x) → y = f(x)
We can also use other symbols in function notation, which are connected to quantities e.g. C(t), where C is cost, t is time in minutes spent on a phone

<u>Ex d</u> For the function $f(x) = x^2 - 2x + 3$, find

1) f(4)	**f(a)**
2) f(0)	**f(2a)**
3) f(-1)	**f(a+5)**

Graphs and function values:

Ex c For the graph find: 1) f(0), and 2) the values of x where f(x) = -1



Ex e Claytons' profit is based on the formula: P(x) = 0.25x – 3, where P(x) = profit, x = # of candies sold.

1) Find the profit of selling 40 candies.

2) Find P(0). What is P(0) in real life?

3) Find the break-even point.

4) Graph the function

Equations: Deciding if an equation is a function

Odd Powers Test – If an equation contains only one y term:

1. If the exponent on y is odd (e.g. y, y^3, y^5…) it is a function
2. If the exponent on y is even (e.g. y^2, y^4, y^6…) it is not a function

Ex f Which of the following equations represent functions?

1) $y^2 = 2x - 3$

2) $y = x^2 + 4x - 1$

3) $x = y^4$

4) $x = y^3$

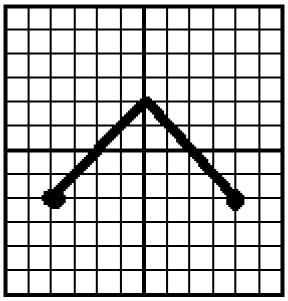
5) $y = x^4$

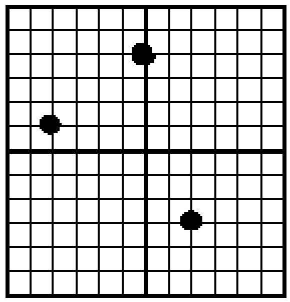
6) $y = x^3$

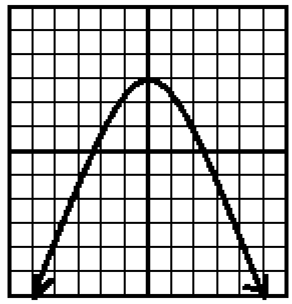
7.2 Domain and Range

Domain – set of all possible x values (inputs)
Range – set of all possible y values (outputs)

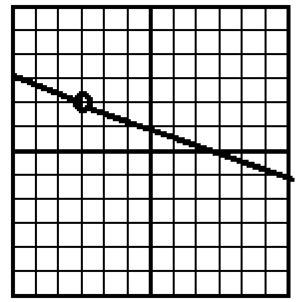
Domain and Range of Graphs







(empty hole)



Some Restrictions on Specific Domains and Ranges

Ex c **Find the domain of** $f(x) = \dfrac{x+1}{x^3 - 4x}$

Ex d **A flare is launched from 224 ft. and its height is described by the equation:**

$h(t) = -16t^2 + 80t + 224$

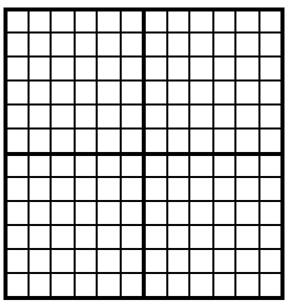
1) Find the value(s) of t when h(t) = 0.

2) Find the domain

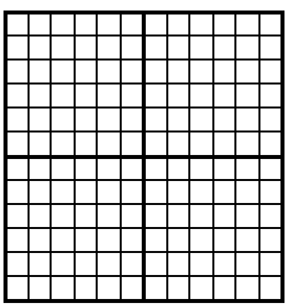
Piecewise functions - use different formulas for different regions

Ex e A phone company charges \$10/month for up to 400 minutes. After 400 minutes, \$0.50 is charged for each additional minute. Write a piecewise function.

Ex f Graph $f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$



Ex g Graph $f(x) = \begin{cases} -x & \text{for } x < 0 \\ x^2 & \text{for } 0 \leq x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

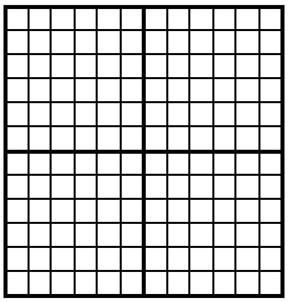


7.3 Graphs of Functions

Linear Functions

Linear functions look like f(x) = ax + b

Ex a Write x + 2y = 6 as a linear function and graph it.



Some special linear functions:

Constant Function: f(x) = k, where k is a fixed number

Ex b My garbage bill:

Identity Function: f(x) = x

Ex c Matching grant:

Ex d A facility rents for $200, and charges $15/meal, represented by the linear function

C(x) = 200 + 15x, where C(x) is the total cost and x is the # of guests

Calculate some ordered pairs and graph the function.

Domain and Range of a Linear Function

Ex d Find the domain and range of:

1) g(x) = - 3x + 2

2) h(x) = 1

Graphing Non-Linear Equations

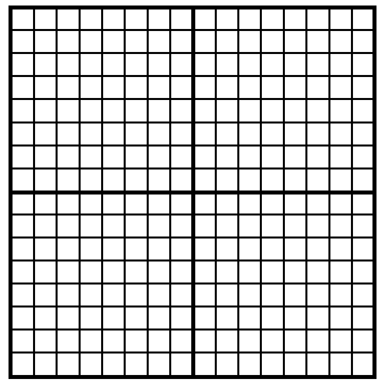
Non-linear equations – don't look like f(x) = ax + b
Some examples:

Polynomial	Absolute value	Rational
$f(x) = x^3 + 1$	$f(x) = \|x\|$	$f(x) = \frac{1}{x}$

They tend to be unpredictable. Graph by plotting points

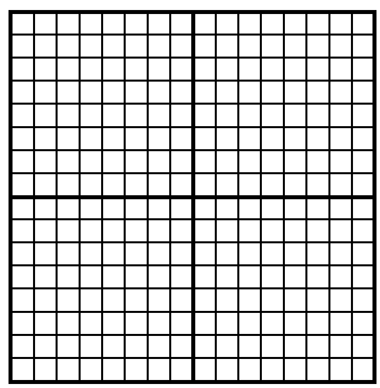
Ex e **Graph $f(x) = x^3 + 1$**

x	f(x)



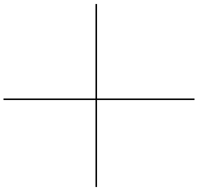
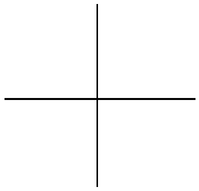
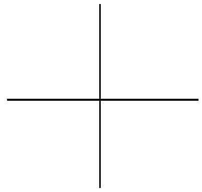
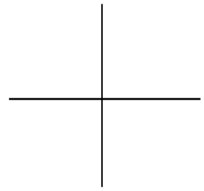
Ex f **Graph $f(x) = \frac{1}{x}$**

x	f(x)

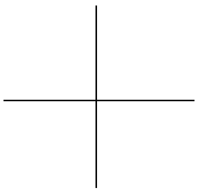
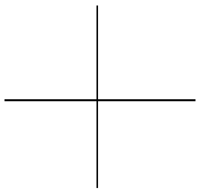
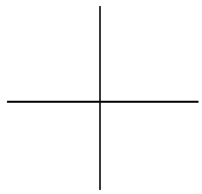
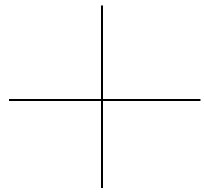


Supplement 2.5 Transformations of Curves

Basic graphs to memorize

1. $f(x) = x$ 2. $f(x) = x^2$ 3. $f(x) = x^3$ 4. $f(x) = x^4$

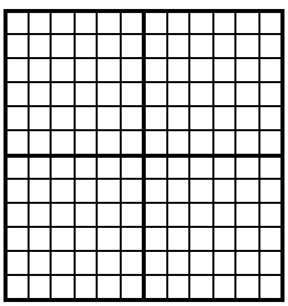





5. $f(x) = |x|$ 6. $f(x) = 1/x$ 7. $f(x) = \sqrt{x}$ 8. $x = y^2$

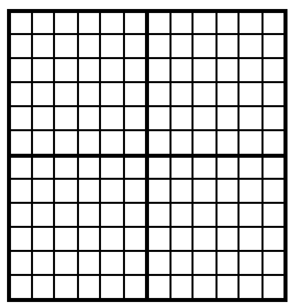
Vertical Translation

$f(x) + k$ shifts $f(x)$ up k units
$f(x) – k$ shifts $f(x)$ down k units

Ex a **$f(x) = x^3 + 3$**



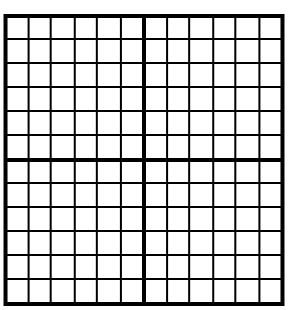
$f(x) = x^3 - 2$



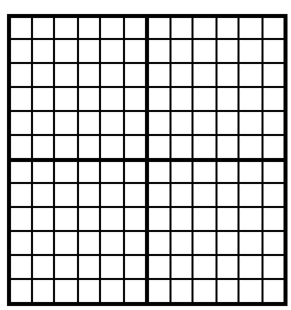
Horizontal Translation

$f(x - h)$ shifts $f(x)$ right by k units
$f(x + h)$ shifts $f(x)$ left by k units

Ex b **$f(x) = (x – 2)^4$**



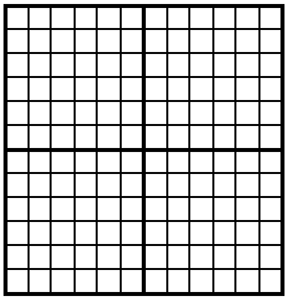
$f(x) = (x + 1)^4$



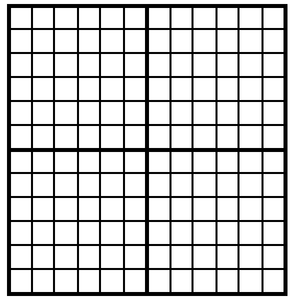
x-axis Reflection

- f(x) reflects f(x) across the x-axis (above/below)

Ex c **f(x) = $-\sqrt{x}$**



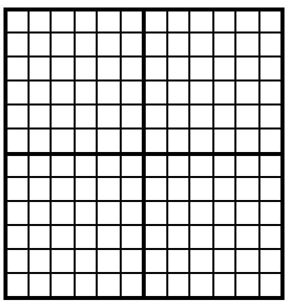
Ex d **f(x) = - |x|**



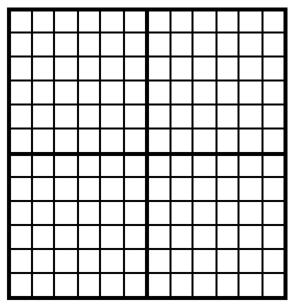
y-axis Reflection

f(-x) reflects f(x) across the y-axis (left/right)

Ex e **f(x) = $\sqrt{-x}$**



Ex f **f(x) = $|-x|$**



Vertical Stretching

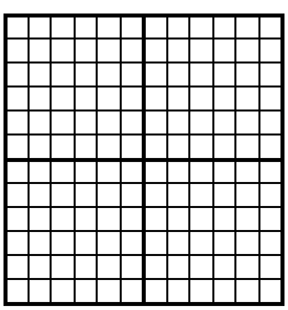
For $c > 1$, $c \cdot f(x)$ stretches f(x) vertically

For $0 < c < 1$, $c \cdot f(x)$ shrinks f(x) vertically

Ex g **f(x) = $2\sqrt{x}$**

g(x) = $\sqrt{x}$

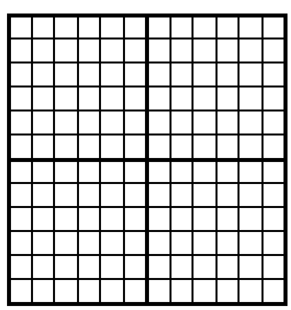
h(x) = $\frac{1}{2}\sqrt{x}$



Ex h **f(x) = $2|x|$**

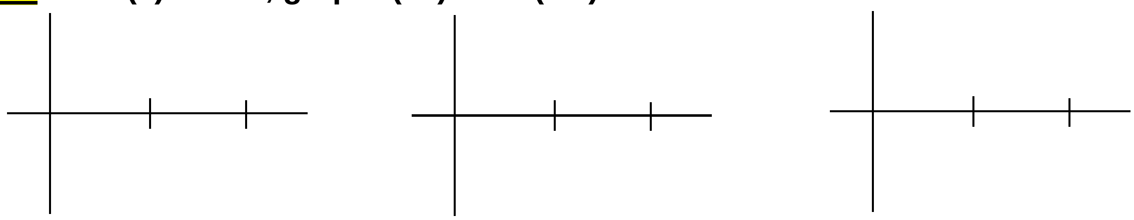
g(x) = $|x|$

h(x) = $\frac{1}{2}|x|$



Horizontal Stretching (somewhat counter intuitive)
For $c > 1$, $f(cx)$ shrinks $f(x)$ horizontally (narrower)
For $0 < c < 1$, $f(cx)$ stretches $f(x)$ horizontally (wider)

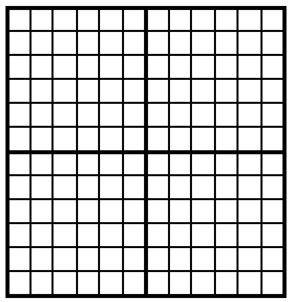
Ex i For f(x) below, graph f(2x) and f(½x)



Ex j $f(x) = \sqrt{4x}$

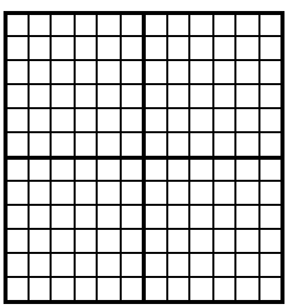
$g(x) = \sqrt{x}$

$h(x) = \sqrt{\frac{1}{4}x}$

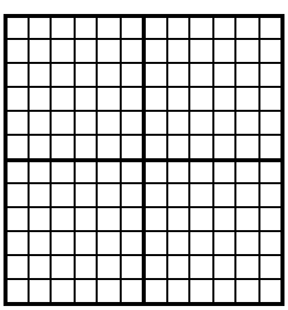


Successive Transformations
1. Do shrinking, stretching, and reflections first
2. Do translations (vertical and horizontal) last

Ex i $f(x) = \sqrt{2 - x} + 3$



Ex j $f(x) = -\frac{1}{2}(x + 1)^3 + 2$



7.4 The Algebra of Functions

Addition, subtraction, multiplication, and division all work as expected

Ex a For $f(x) = x^2 + 6x + 8$ and $g(x) = x + 2$

$(f+g)(x) =$

$(f - g)(x) =$

$(f \bullet g)(x) =$

$(f/g)(x) =$

Ex b For f(x) and g(x) above, find $(f - g)(3)$

Domains and Graphs

1. For (f+g), (f – g), and ($f \cdot g$): Remove bad points from domains of f and g. Remaining points are domain.
2. For (f/g): Remove bad points from domains of f and g. Remove points where denominator, g(x) = 0. Remaining points are domain

Ex c For $f(x) = \frac{2}{x}$ and $g(x) = \frac{x+1}{x-3}$ find the domains of (f+g)(x), (f – g)(x), $(f \bullet g)(x)$, and (f/g)(x)

Ex c For $f(x) = \sqrt{2-x}$ and $g(x) = 2\sqrt{x+3}$

1. **Sketch the graphs**
2. **Find the domains of (f+g)(x), (f – g)(x), $(f \bullet g)(x)$, and (f/g)(x)**

7.5 Applications and Variation

Formulas- Solving for a Specified Variable

Solving for a Specified Variable (procedure)

1. Get rid of denominators (multiply by LCD or cross multiply)
2. Get all terms with desired variable on one side, all other terms on other side
3. Factor out the desired variable
4. Divide by "junk"

Ex a Solve the equation $\frac{d}{r} = \frac{5}{r+2}$ for r.

Ex b Solve the equation $\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$ for r_1 (formula for parallel resistors)

<u>Direct relationship</u> - 2 quantities go up together
Let s(t) = salary , t = # of hours worked

<u>Inverse relationship</u> - one goes up as the other goes down
Let P(x) = inheritance payment, x = # of "kids"

Goal: Get one of 2 formulas, and find a number value for k, to write final formula

- $y = kx$ (direct) OR
- $y = \frac{k}{x}$ (inverse)

<u>Direct Variation</u>

$y = kx$ "y varies as x" or "y varies directly as x"
"y is directly proportional to x"
$y = kx^n$ "y varies directly as the nth power of x"

(first) = k· (second) "first varies as second"
"first is directly proportional to second"

<u>Typical procedure</u> (not all parts are always asked for)

1. Write a generic equation, using "direct" or "inverse" to connect the quantities in the correct relationship. Use k, which represents "varies" or "proportional"
2. Calculate a number for k, using given data.
3. Write a specific equation, exchanging the number for k in the previous equation.
4. Use the specific equation to find the new data point, plugging in the new numbers and solving.

<u>Ex a</u> Distance, d, varies directly as the square of time. A rock falls 64 feet in 2 seconds.
1) Find the proportionality constant, k, and write an equation for d(t)

2) How far does a rock fall in 6 seconds?

Inverse Variation

$y = \frac{k}{x}$ "y varies inversely as x"

"y is inversely proportional to x"

$(\text{first}) = \frac{k}{(\text{second})}$ "first varies inversely as second"

"first is inversely proportional to second"

Ex b The volume of a gas is inversely proportional to air pressure. If the volume is 80 cubic inches at 15 psi, what is the volume at 25 psi?

Joint/Combined Variation

Ex c Drag force on a boat, f, varies jointly as the wet surface area, A, and the square of velocity, v. Write a generic equation connecting these quantities.

Ex d Body Mass Index (B, in this example) varies directly as weight (w) and inversely as the square of height(h).

1. **If B = 23.0 at when h = 70" and w = 160 lb, calculate the proportionality constant, k, and write a formula for B.**

2. **What is the BMI of a 60", 100 lb. girl?**