

Math 90 - Exam 3

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Name Key

When asked to solve, all complex (both real and imaginary) solutions should be given.

1. For the points (4, -3) and (5, 2)
- a) Find the distance between these points

$$d = \sqrt{(5-4)^2 + (2-(-3))^2}$$

$$= \sqrt{1^2 + 5^2} = \sqrt{26}$$

- b) Find the midpoint of these points

$$x = \frac{4+5}{2} = 4.5$$

$$y = \frac{-3+2}{2} = -0.5$$

$$(4.5, -0.5)$$

$$\text{or } (4\frac{1}{2}, -\frac{1}{2})$$

2. Solve:  $\sqrt{15-2x} = x$

$$(\sqrt{15-2x})^2 = x^2$$

$$15-2x = x^2$$

$$0 = x^2 + 2x - 15$$

$$= 0 = (x+5)(x-3)$$

$$x = \cancel{-5}, 3$$

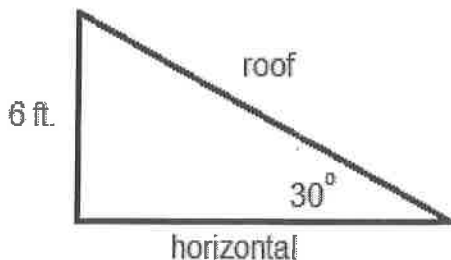
check  $x = -5$  neg  
 pos  $\rightarrow \sqrt{15-2(-5)} = \sqrt{25} = 5$  not sol.

check  $x = 3$

$$\sqrt{15-2(3)} = 3$$

$$\sqrt{9} = 3 \checkmark$$

3. A roof makes a  $30^\circ$  angle with the horizontal beam. The vertical beam is 6 ft. tall.



- a) Find the length of the roof beam

$$a = 6 \text{ ft}$$

$$2a = 12 \text{ ft}$$

- b) Find the length of the horizontal beam.

$$a\sqrt{3} = 6\sqrt{3} \approx 10.4 \text{ ft.}$$

4. Solve:  $3x^2 = x - 5$

$$3x^2 - x + 5 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{-59}}{6} = \frac{1 \pm i\sqrt{59}}{6}$$

5. Solve:  $8(x+1)^2 = 40$

$$\sqrt{(x+1)^2} = \sqrt{5}$$

$$x+1 = \pm \sqrt{5}$$

$$x = -1 \pm \sqrt{5}$$

6. Write an equation whose solutions are  $x = 0$ ,  $x = 3i$ ,  $x = -3i$

$$f(x) = x(x-3i)(x+3i)$$

$$= x(x^2 + 9)$$

$$= x^3 + 9x$$

7. Solve:  $x^4 - 7x^2 + 12 = 0$

$$(x^2 - 4)(x^2 - 3) = 0$$

$$x^2 = 4 \Rightarrow x = \pm 2$$

$$x^2 = 3 \Rightarrow x = \pm \sqrt{3}$$

or let  $u = x^2$

$$u^2 - 7u + 12 = 0$$

$$(u-4)(u-3) = 0$$

$$u = 4 \Rightarrow x^2 = 4, x = \pm 2$$

$$u = 3 \Rightarrow x^2 = 3, x = \pm \sqrt{3}$$

subst. back  $x^2$  for  $u$ .

8. A profit function is represented by the equation:  
 $p(x) = -x^2 + 80x - 300$ , where  $x$  is the number of items sold.

How many items must be sold to maximize the profit?

max: vertex  $x = h = \frac{-b}{2a} = \frac{-80}{2(-1)} = 40$

9. Solve (in interval notation):  $x^2 + 2x - 8 > 0$ .

$(x+4)(x-2) > 0$   
 break pts:  $-4, 2$

	<u>Test</u>	$(x+4)(x-2) > 0$
$x = -6$	$(-)(-) > 0$	T
$x = 0$	$(+)(-) > 0$	F
$x = 3$	$(+)(+) > 0$	T

$(-\infty, -4) \cup (2, \infty)$   
 interval

10. Find the inverse of  $f(x) = 3x - 7$

$y = 3x - 7$   
 switch  $x = 3y - 7$   
 $\frac{x+7}{3} = \frac{3y}{3}$

$y = f^{-1}(x) = \frac{x+7}{3}$

11. For  $f(x) = 2x + 5$ , and  $g(x) = 3x^2$

a) Find  $f \circ g(x)$

$$f(g(x)) = 2[3x^2] + 5$$

$$= 6x^2 + 5$$

b) Find  $g \circ f(-2)$

$$g(f(-2)) = 3[f(-2)]^2 = 3 \cdot [1]^2 = 3 \cdot 1 = 3$$

$f(-2) = 2(-2) + 5 = 1$

12. Solve:  $\frac{1}{27} = 3^{2x-1}$

$$\frac{1}{3^3} = 3^{2x-1}$$

$$3^{-3} = 3^{2x-1}$$

$$-3 = 2x-1$$

$$\frac{-2}{2} = \frac{2x}{2}$$

$$-1 = x$$

$$x = -1$$

13. a) Solve for x:  $\ln(x) = 0.493$

$$e^{\ln x} = e^{0.493}$$

$$x = e^{0.493} = 1.637$$

b) Solve  $x = \log_7 39.6$

$$x = \frac{\log_{10} 39.6}{\log_{10} 7} \approx \frac{1.598}{0.845} = 1.89$$

14. Solve:  $\log_2 x + \log_2(x-2) = 3$

$\log$  term  $\rightarrow \log_2 x(x-2) = 3$   $\leftarrow$  no log

convert to exp:  $2^3 = x(x-2)$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$x = 4, \cancel{x = -2}$$

check  $x = 4$

$$\log_2 4 + \log_2(2) = 3 \checkmark$$

$$2 + 1 = 3 \checkmark$$

check  $x = -2$

$\log_2(-2)$  no good

15. Solve:  $|2e^x| = 15.8$

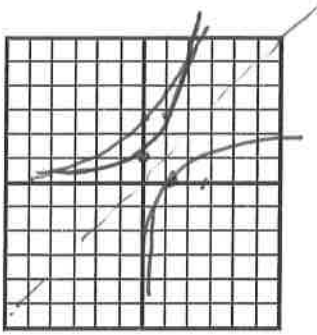
$$e^x = \frac{15.8}{2}$$

$\leftarrow$  take ln of both sides

$$\frac{\ln e^x}{\ln} = \frac{\ln 7.9}{\ln}$$

$$x = \ln 7.9 = 2.069$$

16. Graph:  $f(x) = e^x$  and  $g(x) = \ln x$  on the same graph.



Bonus: Describe the relationship of  $f(x)$  and  $g(x)$  in relation to the line  $y = x$ . Why do these functions have this relationship?

They are symmetrical ~~along~~ about  $y = x$  because they are inverse functions

17. If \$1000 is invested at 7% interest, how much is it worth in 1 year compounded  
a) annually?

$$A = P(1 + r)^t$$

$$A = 1000(1 + .07)^1 = 1070$$

c) continuously?  $A = Pert$

$$A = 1000 e^{.07 \times 1}$$

$$A = 1000 e^{.07} =$$

Bonus: What is the APR of 7% compounded continuously?

