1.1 Intro to Algebra: Expressions, Equations, Translating

Variables - needed for quantities that change

K, $K+2$, $K+4$, $K+6$, $K+9$, $K+12$, $K+15$, $K+18$

Evaluating Expressions – plug in a number for each variable

Ex a Evaluate p^2-2p+1 for $p=5$

$5^2-2(5)+1=25-10+1=16$

No operator between symbols implies mult.

Ex b The area of a triangle is A = ½ bh. Find the area if the base is 6 inches and height is 9 inches.

$A=\frac{1}{2}(6\text{ in})(9\text{ in})=\frac{1}{2}\cdot\frac{6}{1}\cdot\frac{9}{1}\text{ in}^2=27\text{ in}^2$

Translating English to Algebra

4 important words:

sum $+$ difference $-$ product $\cdot$ quotient $\frac{}{}$

Addition
- the sum of a number and 2 $x+2$
- 5 more than a number $x+5$
- 3 added to t $t+3$

Subtraction
- the difference of 3 and a number $3-n$
- 3 less than y $y-3$
- 7 subtracted from a number $n-7$
- x decreased by 8 $x-8$

Multiplication
- the product of 2 numbers xy
- half of x $\frac{1}{2}x$
- twice a number $2n$

Division
- the quotient of a number and 11 $\frac{n}{11}$
- 4 divided by a number $\frac{4}{n}$
- 4 divided into a number $n/4$

Equal is, equals, same as, yields, gives, results in

Ex c Translate to an equation: Six less than twice the product of 4 and a number is 7.

$2\cdot(4n)-6=7$ or $8n-6=7$

1.2 Commutative, Associative & Distributive Laws

First 2 properties work ONLY for addition & multiplication

1. Commutative - you can change order without changing value
   a. Add $a+b = b+a$ $\quad 2+6 = 6+2$
   b. Mult. $a \cdot b = b \cdot a$ $\quad 3(-4) = -4(3)$

Doesn't work for subtraction or division $\quad 3-5 \neq 5-3$

But $3+(-5) = -5+3$

2. Associative – you can change grouping without changing value
   a. Add $(a+b)+c = a+(b+c)$ $\quad (3+4)+5 = 3+(4+5)$
   b. Mult. $(a \cdot b)c = a \cdot (b \cdot c)$ $\quad (2 \cdot 3)(4) = 2(3 \cdot 4)$
   $6 \cdot 4 = 2 \cdot 12$
3. Distributive:
   $a(b+c) = ab+ac)$
   $(a+b)c = ac+bc$
   Triple a recipe
   $3(3E+2F+1S)$
   $= 9E+6F+3S$
4. Identity – element that "does nothing"
   a. Add (1.5) $a+0 = a$ $\quad$ zero is adding identity
   b. Mult. (1.3) $a \cdot 1 = a$ $\quad$ 1 is mult. identity
5. Inverse – element that "undoes" an operation (gets back to identity)
   a. Add $a+(-a) = 0$ $\quad$ $-a$ is add. inverse
   b. Mult. $a \cdot \left(\frac{1}{a}\right) = 1$ $\quad$ $\frac{1}{a}$ is mult. inverse

Note: "Nothing left" in multiplication is NOT zero. $\quad 5 \cdot 1/5 = \frac{5}{5} = 1$

Examples (problem solving) – Simplify, tell property used

Exa $3a - 5b - 7a \overset{\text{comm.}}{=} 3a - 7a - 5b \overset{\text{dist.}}{=} (3-7)a - 5b = -4a - 5b$

Exb $(3x)(4x^2) \overset{\text{assoc.}}{=} 3 \cdot x \cdot 4 \cdot x^2 \overset{\text{comm.}}{=} 3 \cdot 4 \cdot x \cdot x^2 = 12x^3$

Exc $\left(\frac{1}{3}y\right)(3) \overset{\text{comm, assoc}}{=} \frac{1}{3} \cdot 3y = \frac{3}{3}y = y$

Factoring (reverse distributive law) – write as a product

Ex e Write as a product

$5x + 5y = 5(x+y)$

$5a + 15b + 20c = 5(a + 3b + 4c)$

Important distinctions:

Term - an added/subtracted piece. So the expression 3x + 2 has 2 terms. $3x, 2$

Factor - a multiplied piece. So the expression 4ac has 3 factors. $4, a, c$

Ex f List the terms of $x^2 + 5x + (-7)$ → same as $x^2 + 5x - 7$

$x^2, 5x, -7$

Ex g List the factors of $4x(y - 2)$

$4, x, y-2$

Note: Addition/subtraction separates pieces
Multiplication "glues" them together

1.3 Fraction Notation

greater than 1

prime number - a number that can't be "broken down" to smaller factors

e.g. $2, 3, 5, 7, 11 \ldots$

prime factors - the smallest broken down pieces

e.g. $24 = 4 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$

lowest terms – a fraction with no common factors in numerator & denominator

e.g. $\frac{5}{10} = \frac{1}{2}$

not lowest lowest

Multiplying – smart way: cancel common factors first, then multiply nums. & denoms.

Ex a $\frac{18}{25} \cdot \frac{15}{\cancel{27}\,9} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5}$ Dumb way $\frac{270}{225} = \frac{6}{5}$

Reciprocal (mult. Inverse) – flip fraction Don't say cross cancel

recip. of $\frac{2}{5} \rightarrow \frac{5}{2}$, $6 \rightarrow \frac{1}{6}$, $-\frac{1}{4} \rightarrow -4$

Dividing – invert second fraction and multiply (keep, change, flip)

Ex b

$-\frac{\cancel{21}}{35} \div \frac{-27}{20} = -\frac{\cancel{21}}{35} \cdot \left(-\frac{20}{27}\right) = + \frac{\cancel{7}^{\,1}}{\cancel{5}} \cdot \frac{\cancel{20}^{\,4}}{\cancel{27}_{\,3}} = \frac{4}{3}$

Adding and Subtracting Fractions – must have same denominator

Ex c $\frac{5}{8} + \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

If denominators differ, must make a common denominator (LCD)

- If the denominators have no common factors, the LCD is the product of the denominators's, e.g. $\frac{3}{8}$ and $\frac{2}{5}$ →LCD: 40
- If one number is a perfect multiple of all other denominators, that largest number is the denominator, e.g. $\frac{1}{2}, \frac{5}{16}$ and $\frac{3}{4}$ →LCD: 16
- Otherwise, listing (or "eyeballing") multiples of both denoms. can give the LCD, e.g. $\frac{7}{12}$ and $\frac{11}{20}$ 12: 12, 24, 36, 48, 60 20:
- Prime factorization method of finding LCD's need for denoms. with many factors
  - Find prime factors
  - Write all factors of one denominator
  - Find missing factors of other denominators (choose highest powers)

1.4 Positive and Negative Real Numbers

Number systems you should know:
Whole Numbers = {0, 1, 2, 3…}

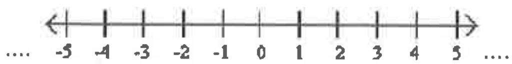
Integers = {…-3, -2, -1, 0, 1, 2, 3…}

Rational Numbers = {x | x is the quotient of 2 integers, a/b, b $\neq$ 0} $2/3$, 3

Irrational Numbers = {x | x is real, but not rational} $\sqrt{2}$, π

Real Numbers = {x | x is a point on the number line} $2/3$, 3, $\sqrt{2}$, π

The Number Line



Smaller   Larger

Exa Write the correct symbol >, <,

$-5 < -2$

$3.27 < 3.7$

Absolute Value -- units from a number to 0 (always positive)

Ex b $|-13| = 13$

$|52| = 52$

Exc $-|46| = -46$

Exd $|9-12| - |-7|$

don't separate single #

$|-3| - 7 = 3 - 7 = -4$

Why abs. value? 2 brothers aged 7 & 12