

R.4 Divisibility Rules

Divisibility Rules – A number is divisible by

- 2 if its ones digit is even
- 3 if sum of digits is divisible by 3
- 6 if its div. by 2 and 3
- 9 if the sum of digits is divisible by 9
- 10 if it ends in 0
- 5 if it ends in 0 or 5
- 4 if the last 2 digits are div. by 4
- 8 if " " 3 digits " " " 8

Ex a Which numbers are divisible by 2?

17	4,201,122	3801	50,000
no	yes	no	yes

Ex b Which numbers are divisible by 3?

29 no	4,201,122 yes	3801 yes	50,000 no
2+9=11 no	4+2+1+1+2+2=12	3+8+1=12	5+0=5

Ex c Which numbers are divisible by 6?

29 no	4,201,122 yes	3801 no	50,000 yes
	yes		no

Ex d Which numbers are divisible by 9?

387 - yes	4,201,122
3+8+7=18	4+2+0+1+2+2=12

Ex e Which numbers are divisible by 10? Which are divisible by 5?

295	3,729,231	1620
yes by 5, no by 10	no by 5, no by 10	yes by 5, yes by 10

Ex f Which numbers are divisible by 4? Which are divisible by 8?

9024	387,231	420
yes by 4	no by 4	yes by 4
" " 8	" " 8	no by 8

1.1 Place Value

A **digit** is **one** of the numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

A **number** – may have several digits, for example 367

1, -12, 495, $\frac{2}{3}$, 2.97, $58\frac{1}{4}$

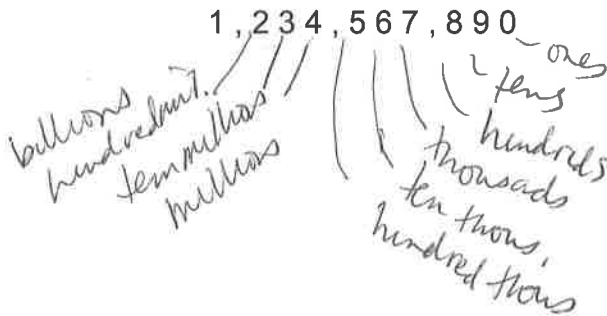
Examples of whole numbers:

1, 495

Examples of non-whole numbers:

-12, $\frac{2}{3}$, 2.97, $58\frac{1}{4}$

Place Values



Use the number 528,174,326,097

Ex a For the number above, what digit is in the hundred thousands place?

3

Ex b What digit is in the ten billions place?

2

Ex c What does the digit "0" represent in the number above?

0 hundreds

Ex d What does the digit "8" represent in the number above?

8 billions

Ex e Write "two million, three hundred fifty thousand, sixty seven" using digits.

2,350,067

Ex f Write 56,204 in word form.

Fifty six thousand, two hundred four

1.2 Introduction to Integers

Integers – positive and negative whole numbers.

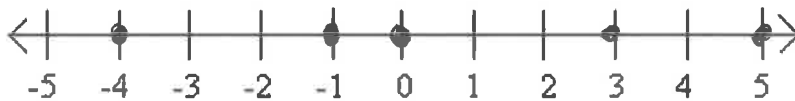
Examples:

withdraw \$60, deposit \$200, 10 degrees below zero, 70 degrees

-60 $+200$ -10° 70°

Note: If a number has no sign:

Ex a Represent the following numbers on the number line: 3, -1, 0, -4, 5



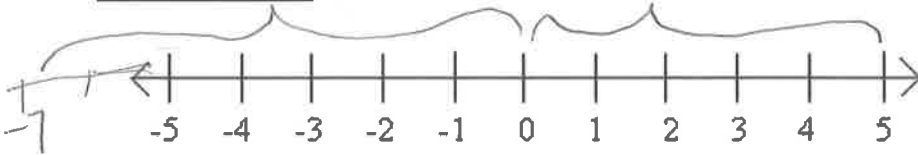
Ex b Use $<$ or $>$ to write a true statement for each pair

$$3 > -8$$

$$0 > -3$$

$$-12 < -5$$

Absolute Value – the distance from a number to 0 on a number line:



$$|5| = 5$$

$$|-7| = 7$$

$$|0| = 0$$

If a number is positive, the absolute value is positive

If a number is negative, the absolute value is positive

Ex c Simplify:

$$|-68|$$

$$68$$

$$|349|$$

$$349$$

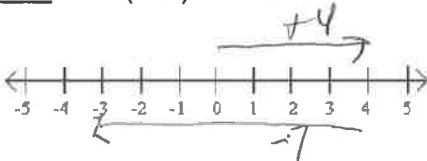
$$|-10,000|$$

$$10,000$$

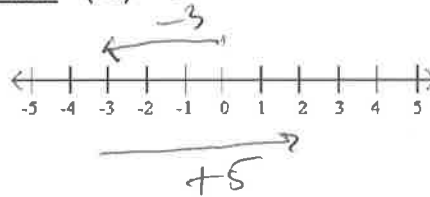
1.3 Adding Integers

Analogies: gaining or losing yards in football, depositing or withdrawing money

Ex a $4 + (-7) = -3$



Ex b $(-3) + 5$



Ex c $-3 + (-6) = -9$



Adding without number lines:

- Add 2 numbers with same sign (both positive or both negative)
 - Add absolute values (amounts)
 - Use the common sign
- Add 2 numbers with opposite signs (1 positive, 1 negative):
 - Subtract the amounts, with the larger abs. value on top
 - Use the sign of the "dominant" number (greater absolute value)

Ex d Add the integers

$$\begin{array}{r} -30 + (-14) \\ \underline{-30} \\ -14 \\ \hline -44 \end{array}$$

$$\begin{array}{r} -11 + 20 \\ \underline{20} \\ -11 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 13 + (-41) \\ \underline{-41} \\ 13 \\ \hline -28 \end{array}$$

Properties of Addition

- Commutative - you can change order without changing value
- Associative - you can change grouping without changing value

$$\begin{array}{l} 3 + 5 = 5 + 3 \\ -2 + 6 = 6 + (-2) = -16 \\ (2 + 3) + 4 = 2 + (3 + 4) \\ 5 + 4 = 2 + 7 \\ 9 = 9 \end{array}$$

- Identity Property of Addition - element that makes no change

The identity property is sometimes called the zero property because adding 0 to a number leaves it unchanged

$$\underline{0} + 21 = 21$$

$$-42 + \underline{0} = -42$$

zero is the additive identity

Adding More than 2 Integers

Method 1: Work left to right

Ex e $-2 + 7 + (-3) + 6 + (-5)$

$$\underbrace{-2 + 7}_{5} + (-3) + 6 + (-5) = 2 + 6 + (-5) = 8 + (-5) = 3$$

Method 2: Group all positives together, and all negatives together. Add both groups.

<u>Pos</u>	<u>Neg</u>	
7	-2	
6	-3	
13	-5	
	-10	= 3 (pos)

Strategic grouping: If you have opposites that cancel, or numbers that make easier sums, use them!

$$\text{Ex f } 9 + (-31) + (-18) + 31 + (-6) = 9 + (-31) + 31 + (-18) + (-6)$$

$$9 + 0 + (-18) + (-6) = -9 + (-6) = -15$$

$$\text{Ex g } -2 + (-14) + (-98)$$

$$-100 + (-14) = -100 + (-14) = -114$$

Commutative

Practice Problems

1. $-13 + (-41) + 13 + (-19)$

$$= -41 + (-19) = -60$$

2. $-46 + 71 + (-4) + (-16)$

$$= -50 + 71 + (-16) = 21 + (-16) = 5$$

3. $-34 + (-18) + 18 = -34$

4. $-7 + 28 + (-11) + (-13) + 16$

<u>Pos</u>	<u>Neg</u>	
28	-7	
16	-11	
44	-13	
	(-31)	= 13

1.4 Subtracting Integers

Opposites (Additive Inverses)

Ex a Find the opposites of the numbers: 17 - 49 0

$$-17 49 0$$

The sum of a number and its opposite is 0

Subtraction Procedure: (2 changes that cancel)

1. Change (-) symbol to (+) symbol.
2. Change the sign of the second number

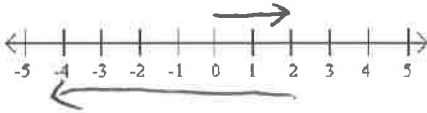
Subtract negative number \rightarrow add pos

$(-)(-)$ \rightarrow $(+)(+)$

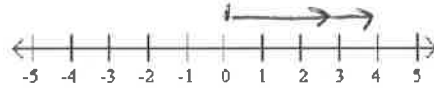
Subtract positive number \rightarrow add neg

$(-)(+)$ \rightarrow $(+)(-)$

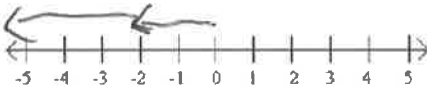
Ex b $2 - 6 \rightarrow 2 + (-6) = -4$



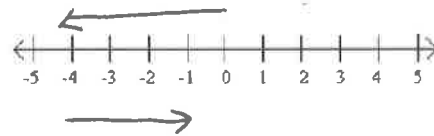
Ex c $3 - (-1) = 3 + +1 = 4$



Ex d $-2 - 3 = -2 + (-3) = -5$



Ex e $-4 - (-3) = -4 + +3 = -1$



Ex f $5 - |-7| - (11 - 3)$

$$5 - 7 - (8) = -2 - 8 = -10$$

Practice Problems

1. $6 - 11 = -5$

2. $-8 - 5 = -13$

3. $0 - 41 = -41$

4. $-18 - (-30) = 12$

5. $-3 - |-6| - 5 = -3 - 6 - 5$
 $= -14$

6. $19 - (11 - 7) + 1$
 $19 - (4) + 1 = 15 + 1 = 16$

1.5 Rounding and Estimating

Ex a Round 29 to the nearest 10 $\rightarrow 30$

Round - 22 to the nearest 10 $\rightarrow -20$

Round - 25 to the nearest 10 $\rightarrow -30$

Rounding Whole Numbers Procedure – for a specific place

1. Find the digit in the specified place.
2. Look at the digit AFTER that place

3. If the “after” digit is 5 - 9 round up

If the “after” digit is 0 - 4 keep same (keep lower number)

4. Replace the rounded digits with zeros

5. **Keep all digits to the left of the specified place**

Ex b Round 3,682,357 to the nearest:

million	ten thousand	hundred	ten
4,000,000	3,680,000	3,682,400	3,682,360

Front-End Rounding- Keep one non-zero digit on the front, replace other digits with 0

Ex c Use front-end rounding to estimate the following amounts

Restaurant bill: \$ 43.58

Truck: \$27,875

House: \$239,995

\$ 40.

30,000

200,000

240,000 is not front-end

For Examples below, estimate using front-end rounding. Then find the exact values rounding

Ex d $58 + 91 + 37 \approx 186$

$\approx 60 + 90 + 40 = 190$

Ex f $829 + 2640$

$$\begin{array}{r} 800 \quad 829 \\ + 3000 \quad 2640 \\ \hline 3800 \quad 3469 \end{array}$$

Ex e $764 - 238$

$$\begin{array}{r} \approx 800 \quad 764 \\ - 200 \quad - 238 \\ \hline 600 \quad 526 \end{array}$$

Ex g $\$492 - \$61 + \$88 - \179

$\approx 500 - 60 + 90 - 200 = 330$

$$\begin{array}{r} 492 \quad -61 \\ 88 \quad -179 \\ \hline 580 \quad -240 \\ = 340 \end{array}$$

Note: Front-end rounding works best when the highest place values are similar (differ by 1 or less)

doesn't work: $239,995 + 2.13$

$$\begin{array}{r} \rightarrow 200,000 \\ \hline 200,002 \end{array}$$

1.6 Multiplying Integers

Product of a positive and a negative number – the result is Negative

Tip: Determine the sign and set it aside, then multiply the absolute values separately.

Ex a $7(-6) = -42$ $(-3)(5) = -15$ $(-11) \cdot (28) = -308$

Product of 2 negative numbers – the result is positive

Ex b $(-5)(-3) = +15$ $-2(-7) = \oplus 14$ $(-25) \cdot (-12) = \oplus 300$

Properties of Multiplication

1. Commutative - you can change order without changing value

$3 \cdot 5 = 5 \cdot 3$ Does not work for dw. $\frac{4}{2} \neq \frac{2}{4}$

2. Associative – you can change grouping without changing value

$(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$ // Does not work for division
 $12 \cdot 5 = 3 \cdot 20$ $(12 \div 4) \div 2 \neq 12 \div (4 \div 2)$
 $60 = 60$ $3 \div 2 = \frac{3}{2} \neq 12 \div 2 = 6$

3. Identity Property of Multiplication – element that results in no change

The identity property of multiplication is sometimes called the Multiplication Property of 1 because multiplying 1 by a number leaves it unchanged

$1 \cdot 5 = 5$ or $17 \times 1 = 17$

4. Multiplication Property of 0 - multiplying by 0 gives a product of 0

$$6 \cdot 0 = 0$$

$$0(47) = 0$$

5. Distributive

$$a(b + c) = ab + ac$$

$$(a + b)c = ac + bc$$

Similar to doubling a recipe:

$$2(3E + 2F + 1S) = 6E + 4F + 2S$$

Multiplying More than 2 Numbers

Every pair of multiplied negative numbers produces a positive number.

For more than 2 negatives:

An even # of negatives mult. produces a positive number.

An odd # of negatives mult. produces a negative number.

Exc $(-4)(-3)(2)(-1) = -24$

Exc d $(-5)(4)(-3)(-2)(-1) = (+) 120 = 120$

Exc e $2(-47)(-5) = (+) 47 \cdot 10 = 470$

Exc f Write an integer in each blank to make a true statement:

$$-8 \times (\underline{3}) = -24$$

$$35 = (\underline{-5})(-7)$$

Exc g Each month, \$38 is deducted from a bank account.

1) Estimate the amount deducted in a year

$$12 \times 38 \approx 10 \times 40 = \$400 \text{ deducted}$$

or $-\$400$

2) Calculate the exact amount deducted in a year

$$\begin{array}{r} 12 \\ \times 38 \\ \hline 96 \\ 360 \\ \hline 456 \end{array}$$

$\$456$ ded. or $-\$456$

1.7 Dividing Integers

The rules for signs in division are the same as for multiplication.

Quotient of a positive and a negative number – the result is negative

Ex a $\frac{-30}{5} = -6$

$$\frac{27}{-9} = -3$$

$\frac{\text{pos}}{\text{neg}}$ or $\frac{\text{neg}}{\text{pos}}$

Quotient of 2 negative numbers – the result is positive

Ex b $\frac{-42}{-6} = 7$

$$-48 \div (-12) = 4$$

Recall Division properties

1. $(-9) \div 1 = \frac{-9}{1} = -9$

2. $(-28) \div (-28) = \frac{-28}{-28} = 1$

3. $0 \div (-6) = \frac{0}{-6} = 0$

4. $-11 \div 0 = \frac{-11}{0} = \text{undefined}$

Why?

Related equations: $\frac{-24}{3} = -8$

$$\rightarrow -24 = -8(3)$$

$$\frac{-24}{0} = x$$

$$\rightarrow -24 = x \cdot 0 \text{ impossible}$$

$$\frac{0}{-24} = x$$

$$\rightarrow 0 = x \cdot (-24); \quad x=0 \text{ possible}$$

Multiplication and Division together

Ex d $8(-50) \div (-2 \times 5)$ parentheses first

$$-400 \div (-10) = \frac{-400}{-10} = 40$$

Applications

Ex e A house needs 300 ft. of baseboard. Baseboard is sold in 8-ft pieces. How many pieces must be purchased?

$$8 \overline{) 300} \begin{array}{r} 37 \\ -24 \\ \hline 60 \\ -56 \\ \hline 4 \end{array} \quad \frac{4}{8} = \frac{1}{2} \rightarrow \text{buy } 38$$

Ex f If a car loan (with interest) is \$17,400 is made over 5 years, how many monthly payments must be made? Estimate the amount of each payment. Then calculate the exact amount of each payment to the nearest dollar.

$$5 \text{ years} \times \frac{12 \text{ months}}{\text{year}} = 60 \text{ months (payments)}$$

$$\text{Estimate: } \frac{20,000}{60} = \frac{2000}{6} \quad \begin{array}{r} 333 \frac{2}{3} \\ 6 \overline{) 2000} \\ \underline{18} \\ 20 \end{array} = 333 \frac{2}{3} \quad \text{Not easy}$$

$$\frac{18,000}{60} \approx 300/\text{month} \text{ each}$$

$$\text{Exact } 6 \overline{) 17400} \begin{array}{r} 290 \\ \underline{12} \\ 54 \\ \underline{48} \\ 60 \\ \underline{60} \\ 0 \end{array} \text{ exact: } \$290/\text{month}$$

Practice Problems

1. Simplify: $\frac{0}{-19} = 0$ $\frac{23}{-1} = -23$ $\frac{-52}{0}$ undefined $\frac{-87}{-87} = 1$

2. Simplify: $(-24 \div 6)(18 \div (-2))$

$$\underbrace{(-24 \div 6)}_{-4} \underbrace{(18 \div (-2))}_{-9} = 36$$

3. How many \$3 sodas can be bought with a \$50 bill?

$$3 \overline{) 50} \begin{array}{r} 16 \frac{2}{3} \\ \underline{48} \\ 20 \end{array} \rightarrow \text{can buy } 16$$

1.8 Exponentials and Order of Operations

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 4 \cdot 4 = 16$
exponent form *standard/place value*

$3^3 = 3 \cdot 3 \cdot 3 = 27$ $(-3)^3 = (-3)(-3)(-3) = -27$ $-3^3 = -3 \cdot 3 \cdot 3 = -27$

$5^2 = 5 \cdot 5 = 25$ $(-5)^2 = (-5)(-5) = 25$ $-5^2 = -5 \cdot 5 = -25$

Handwritten notes:
 In 2^4 , the 2 is circled and labeled "base", and the 4 is circled and labeled "exponent".

Ex a Write in exponent form: $7 \cdot 7 \cdot 7$

$$7^3$$

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$$10^6$$

Ex b Evaluate: $7 \cdot 7 \cdot 7$

$$49 \times 7 = 343$$

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$$1,000,000$$

Ex c Evaluate $(-2)^3(-7)^2$

$$(-2)(-2)(-2) \cdot (-7)(-7)$$

$$= (-) 8 \cdot 49 = -392$$

Simplifying Expressions (Order of Operations for several operations)

1. Parentheses (and grouping symbols like $\{ \}$ or $[]$)
2. Evaluate all exponential expressions
3. Multiplication and Division, in order from left to right
4. Addition and Subtraction, in order from left to right

Ex c $100 - (68 - 21)$

$$100 - 47 = 53$$

$(100 - 68) - 21$

$$32 - 21 = 11$$

Ex d $5 \cdot 2^2$

$$5 \cdot 4 = 20$$

$(5 \cdot 2)^2$

$$(10)^2 = 100$$

2.1 Introduction to Variables

We represent real quantities as variables, because these quantities keep changing.

My husband's age: K $K+9$
 $K+2$ $K+12$
 $K+4$ $K+15$
 $K+6$ $K+18$

variable - letter representing quantity that changes

constant - number without variable

expression - combination of numbers and variables $K+2$

coefficient - number multiplying variable

Ex a Suppose your job pays \$20/hour, but the number of hours you work changes from week to week. Write an expression using the variable "h", which calculates the salary earned from working "h" hours.

$$20h$$

h is variable

20 is coefficient

$$20h + 6$$

↑ ↑
coefficient constant

Evaluating an expression - "Evaluate" means "find the number value"

Ex b Evaluate the expression above for $h = 17$

$$20(17) = 340 \quad \text{operation is multiplication}$$

Ex c If 4 more photocopies are made than the number of students:

- 1) Choose a variable for the number of students, and write an expression for the number of copies needed.

$$s + 4$$

- 2) If there are 38 students in the class, how many copies are made?

$$38 + 4 = 42$$

- 3) If there are 44 students, how many copies are made?

$$44 + 4 = 48$$

Ex d Evaluate the expression $2s + 5b$ for $s = 10$, $b = 7$

$$2(10) + 5(7)$$

$$20 + 35 = 55$$

sodas cost \$2 each

burgers " \$10 each

total cost of 10 sodas, 7 burgers

Properties Using VariablesCommutative Prop of Addition: $a + b = b + a$

$$3 + (-5) = -5 + 3$$

Commutative Prop. of Multiplication: $ab = ba$

$$3(-5) = (-5) \cdot 3$$

Exponents with variables

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

$$x^4 = x \cdot x \cdot x \cdot x$$

$$-7x^2y^5 = -7 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$$

Ex d Evaluate $2a^2b$ for $a = -3$, $b = 2$

$$2(-3)^2(2) = 2 \cdot 9 \cdot 2 = 36$$

Ex e Evaluate $|xy - z|$ for $x = -5$, $y = 3$, $z = 7$

$$|(-5)(3) - 7| = |-15 - 7| = |-22| = 22$$

Try Evaluate $5a^2 - 4b$ for $a = -2$, $b = -7$

$$5(-2)^2 - 4(-7)$$

$$5 \cdot 4 + 28 = 20 + 28 = 48$$

2.2 Simplifying Expressions

term – an added or subtracted “piece” in an expression. It may contain numbers, variables, and/or multiplication (glue)

Examples of single terms:

$$-3, 4x, y^5, -6xy^3z$$

Examples of things that are NOT single terms:

$$x+2, y^2-7y+12$$

A variable term has 2 parts:

1. coefficient - the number part, including the sign
2. variable part - the letters, including exponents

Exa Find the coefficient and variable part of each term below:

$$3x^2y \quad -x \quad y^5 \quad -4$$

C: 3 VP: x^2y C: -1, VP: x C: 1 VP: y^5 C: -4, no VP

A term without a variable part is called a constant

Like terms - terms that have exactly the same variable parts (including exponents)

Exb Are the following pairs of terms like or unlike?

$$3a \quad ? \quad -7b \quad U$$

$$2x \quad ? \quad x^2 \quad U$$

$$-3xy \quad ? \quad 4yx \quad L$$

$$abc \quad ? \quad -2abc \quad L$$

$$2ab^2 \quad ? \quad 3a^2b \quad U$$

Combining Like Terms – treat as “types”, with amounts of each type

1. Identify each type of variable part
2. For “like” types, add the coefficients. For unlike types, keep separate.

(or subtract)

Exc $4x^2 + x^2 - 2x^2 = 3x^2$

Exd $3a + 7b - 5a + b - 4c = -2a + 8b - 4c$

Exe $-4y^2 - 9y^2 + 2y = -13y^2 + 2y$

A simplified expression should have:

1. No parentheses (use distributive law if needed)
2. Multiplied pieces combined
3. Like terms combined

$$\text{Ex 4} \quad 2(5x - 3y) = 10x - 6y$$

$$\text{Ex 9} \quad -12(-p) = 12p$$

$$\text{Ex 6} \quad (-2x^3)(4x^2) = -8x^5$$

$$\begin{aligned} \text{Ex 1} \quad -2(4b + 11) + 5b &= -8b - 22 + 5b \\ &= -3b - 22 \end{aligned}$$

$$\begin{aligned} \text{Ex 1} \quad 8 + 4(3z + 5) - z \\ = 8 + 12z + 20 - z = 11z + 28 \end{aligned}$$

Practice Problems

Simplify:

$$1. \quad 3x^2 + 2x + 7x^2 = 10x^2 + 2x$$

$$2. \quad (-5x^3)(2x) = -10x^4$$

$$3. \quad 3(4x - y + 5) = 12x - 3y + 15$$

$$4. \quad -2(b + 4) + 7b - 11 = -2b - 8 + 7b - 11 = 5b - 19$$

5. Evaluate $\frac{1}{2}at^2$ for $a = 32$, $t = 3$

$$\frac{1}{2}(32)(3) = 16(3) = 48$$

2.3 Solving Equations Using Addition (or Subtraction)

Solve - get a number that makes the equation true

Solution - a number that makes the equation true

Checking to see if a number is a solution - Procedure

1. Replace the variable(s) with the number
2. Simplify and determine if the equation is true

Exa B $x=10$ a solution of

$$x - 3 = 13 ?$$

$$10 - 3 \neq 13 \quad \text{no}$$

Addition/Subtraction Property of Equality - you can add or subtract the same amount from both sides without changing the solution

$$\text{If } A = B, \text{ then } A + C = B + C$$

$$\text{(also) If } A = B, \text{ then } A - C = B - C$$

Goal in Solving: Isolate the variable

get x by itself
get $x = \text{number}$

Solving Equations with added "clutter"

1. Combine like terms on each side of the equal sign.
2. Look at the variable. Decide what is "clutter"
3. Decide how the "clutter" is connected to the variable. Do the reverse to "undo" the clutter.
4. Isolate the variable
5. Check

Exa solve $x - 30 = 21$

$$\begin{array}{r} x - 30 = 21 \\ +30 \quad +30 \\ \hline x = 51 \end{array}$$

check: $51 - 30 = 21$

$$21 = 21 \quad \checkmark$$

Ex b solve $9 = -17 + y$

$$\begin{array}{r} 9 = -17 + y \\ +17 \quad +17 \\ \hline 26 = y \rightarrow y = 26 \end{array}$$

convention

Ex c solve $3x + 18 - 2x = 11$

$$\begin{array}{r} 3x + 18 - 2x = 11 \\ \hline x + 18 = 11 \\ -18 \quad -18 \\ \hline x = -7 \end{array}$$

check: $3(-7) + 18 - 2(-7) = 11$?

$$\begin{array}{r} -21 + 18 + 14 = 11 \\ -21 + 32 = 11 \quad \checkmark \end{array}$$

2.4 Solving Equations Using Division (or Multiplication)

Multiplication/Division Property of Equality - you can multiply or divide by the same amount on both sides without changing the solution

$$\text{If } A = B, \text{ then } A/C = B/C$$

$$\text{(also) If } A = B, \text{ then } AC = BC$$

"Undo" clutter by doing the reverse operation

$$\begin{array}{l} \text{Ex 9} \quad 9t = 45 \qquad t = 5 \\ \quad \quad \frac{9t}{9} = \frac{45}{9} \end{array}$$

$$\text{Ex 6} \quad \frac{x}{4} = 20 \quad ; \quad \frac{x}{4} \cdot 4 = 20 \cdot 4 \quad , \quad x = 80$$

$$\text{Ex c} \quad -x = 12 \quad ; \quad \frac{-x}{-1} = \frac{12}{-1} \quad ; \quad -x = -12$$

$$\begin{array}{l} \text{Ex d} \quad -14y + 8y = -8 - 62 \\ \quad \quad -6y = -54 \quad ; \quad y = 9 \end{array}$$

$$\text{Ex e} \quad -21p = 0 \quad \frac{-21p}{-21} = \frac{0}{-21} \quad ; \quad p = 0$$

$$\text{Ex f} \quad (-3)(-2x) = 48 \quad ; \quad 6x = 48, \quad x = 8$$

Practice Problems - Solve the following

(from 2.3 and 2.4)

1. $32 + x = 20$

$$x = -12$$

2. $-t + 19 + 2t = -6$

$$t = -25$$

3. $35 = -k$

$$k = -1$$

4. $12 = \frac{y}{3}$

$$y = 36$$

5. $160 - 139 = 10x - 3x$

$$\frac{21}{21} = \frac{7x}{21} \quad ; \quad x = 3$$

2.5 Solving Equations with Several Steps

What if you have both added & multiplied "clutter"?

Ex 9 $3x + 11 = 50$

"Play dumb" - get rid of 3 first

$$\frac{3x}{3} + \frac{11}{3} = \frac{50}{3}$$

Fractious,
ugh!
 $x = \frac{50}{3} - \frac{11}{3}$

Easier way

Get rid of 11 first

$$3x + 11 = 50$$

$$\quad -11 \quad -11$$

$$\frac{3x}{3} = \frac{39}{3}$$

$$\boxed{x = 13}$$

Solving Procedure (summary)

1. Simplify if needed (clear parentheses, combine like terms)
2. Get all variable terms on one side, by adding or subtracting from both sides; combine into one variable term
3. Get rid of added/subtracted "clutter"
4. Get rid of multiplied/divided "clutter"

Variable terms on both sides

Ex 10 $2x - 19 = 4x - 11$

$$\begin{array}{r} -2x \qquad -2x \\ -19 = 2x - 11 \\ +11 \qquad +11 \\ -8 = 2x \\ x = -4 \end{array}$$

Ex 11 $-4x + 17 = x - 13$

$$\begin{array}{r} +4x \qquad +4x \\ 17 = 5x - 13 \end{array}$$

$$17 = 5x - 13$$

$$30 = 5x$$

$$x = 6$$

Clearing Parentheses

Ex 12 $2(4t - 9) = 40 - (t - 5)$

$$8t - 18 = 40 - t + 5$$

$$8t - 18 = 45 - t$$

$$9t - 18 = 45$$

$$9t = 63 \quad ; \quad t = 7$$

Practice Problems

1. $x + 7 + 3x = 5x - 12$

$$4x + 7 = 5x - 12$$

$$7 = x - 12$$

$$19 = x$$

2. $4 - 5(x - 2) = -4(x + 1)$

$$4 - 5x + 10 = -4x - 4$$

$$14 - 5x = -4x - 4$$

$$14 = x - 4$$

$$x = 18$$

3. $5(p - 3) + 6p - 2 = 60$

$$5p - 15 + 6p - 2 = 60$$

$$11p - 17 = 60$$

$$11p = 77$$

$$p = 7$$

4. $-2(x + 7) - 9 = 5x + 12$

$$-2x - 14 - 9 = 5x + 12$$

$$-2x - 23 = 5x + 12$$

 $+2x$ $+2x$

$$-23 = 7x + 12$$

$$-35 = 7x$$

$$x = -5$$

3.1 Perimeter Problems

Perimeter – the distance around the outside of a shape. The operation is: *addition*

Rectangle: has 4 angles which are 90° (right angles) Question: *Is a square a rectangle*

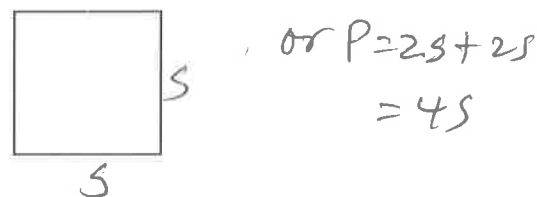
Formula for Perimeter of a rectangle:

$$P = 2L + 2W$$

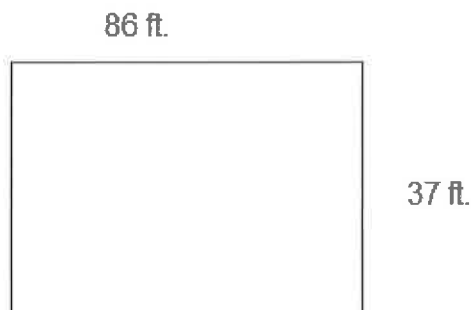


Formula for Perimeter of square:

$$P = 4S$$



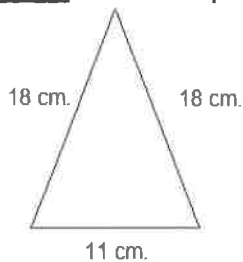
Ex a Find the perimeter of the rectangle:



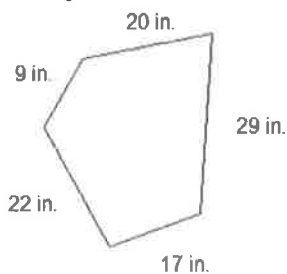
$$\begin{aligned} P &= 2(86 \text{ ft}) + 2(37 \text{ ft}) \\ &= 172 \text{ ft} + 74 \text{ ft} \\ &= 246 \text{ ft} \end{aligned}$$

Irregular objects:

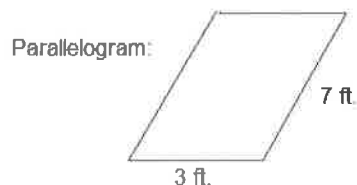
Ex b Find the perimeter of each object:



$$\begin{aligned} P &= 18 \text{ cm} \\ &18 \text{ cm} \\ &\underline{11 \text{ cm}} \\ &47 \text{ cm} \end{aligned}$$

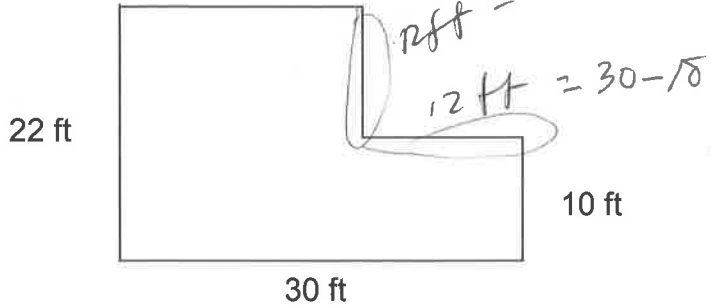


$$\begin{aligned} P &= 20 \text{ in} \\ &29 \\ &17 \\ &22 \\ &9 \\ &\underline{\quad} \\ &97 \text{ in} \end{aligned}$$



$$\begin{aligned} P &= 2(3 \text{ ft}) + 2(7 \text{ ft}) \\ &= 6 \text{ ft} + 14 \text{ ft} \\ &= 20 \text{ ft} \end{aligned}$$

Ex c Find the perimeter:
18 ft



$$\begin{array}{r}
 P = 10 \text{ ft} \\
 + 30 \\
 + 22 \\
 + 18 \\
 + 12 \\
 + 12 \text{ ft} \\
 \hline
 104 \text{ ft}
 \end{array}$$

or

$$P = 2(30) + 2(22) = 60 + 44 = 104 \text{ ft}$$

Ex d A 9' X 10' room is decorated with border paper. If each roll is 12 ft, how many rolls are needed?

$$P = 2(9 \text{ ft}) + 2(10 \text{ ft}) = 18 \text{ ft} + 20 \text{ ft} = 38 \text{ ft}$$

$$\begin{array}{r}
 3 \frac{1}{2} \\
 12 \overline{) 38} \\
 \underline{36} \\
 2
 \end{array}$$

Ex e A living room is 18 ft X 12 ft. The doorway into the living room is 6 ft wide.

a) If baseboard costs \$2.25/foot, what is the cost of installing baseboard?

$$P = 2(18) + 2(12) = 36 + 24 = 60 \text{ ft}$$

total length = 60 - 6 = 54 ft

$$\text{cost} = \$2.25(54) = \$121.50$$

b) If baseboard is only sold in 8-ft segments for \$12 each, what is the cost?

$7 \frac{6}{8} = 6 \frac{3}{4}$ + buy 7 segments

$$\begin{array}{r}
 8 \overline{) 54} \\
 \underline{48} \\
 6
 \end{array}$$

$$\begin{array}{r}
 \$12 \\
 \times 7 \\
 \hline
 \$84
 \end{array}$$

(~~more~~ cheaper by piece)

3.2 Area Problems

Area of a rectangle:



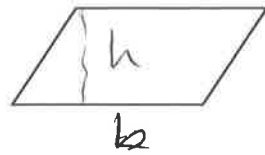
$$A = LW$$

Area of a Square



$$A = S \cdot S = S^2$$

Area of a Parallelogram:



$$A = b \cdot h$$

Ex a The area of a rectangle is 240 ft^2 . The length is 20 ft. Find the width.

$$A = L \cdot W$$

$$240 = 20W$$

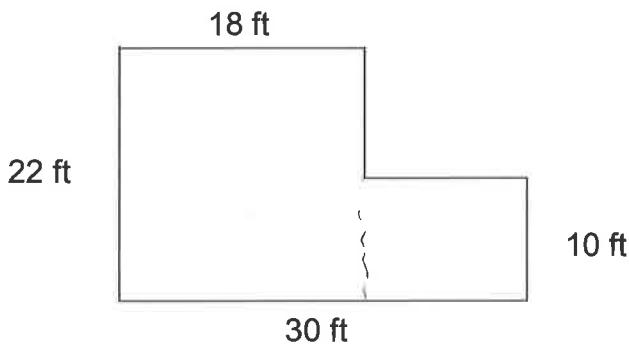
$$12 = W$$

$$W = 12 \text{ ft}$$

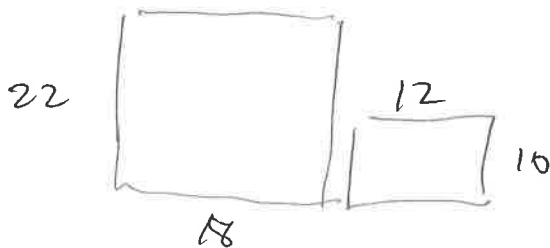
Ex b A parallelogram has ~~height~~ a base of 17 inches and height of 11 inches. Find the area.

$$A = (17 \text{ in}) (11 \text{ in}) = 187 \text{ in}^2$$

Exc Find the area of the room below. If the cost of carpet is $\$4/\text{sq. ft}$, how much does it cost to carpet the room?



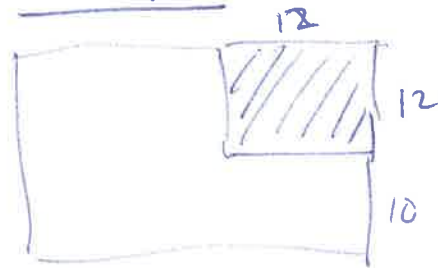
Method 1



$$A = (18)(22) + (12)(10)$$

$$= 396 + 120 = 516 \text{ ft}^2$$

Method 2



$$A = (22)(30) - (12)(12)$$

$$= 660 - 144 = 516 \text{ ft}^2$$

Cost : $(516 \text{ sq. ft})(\$4/\text{sq. ft})$

$$= \$2064$$

Practice Problems

1. a) Find the perimeter of a square picture frame with 9 inches on each side.

$$P = 4s = 4(9 \text{ in}) = 36 \text{ in}$$

- b) Find the area of the picture in the frame.

$$A = s^2 = (9 \text{ in})^2 = 81 \text{ in}^2$$

2. A yard is enclosed with chicken wire fencing. If each roll of 50 ft. costs \$26, how much does it cost to enclose a 60 ft X 30 ft back yard?

$$P = 2(60) + 2(30)$$

$$= 120 + 60 = 180 \text{ ft}$$

$$\# \text{ of rolls} = \frac{180}{50} = 3\frac{3}{5} \rightarrow \text{buy } 4$$

$$\text{cost} = \$26 \times 4 = \$104$$

3. A room is 12 ft X 15 ft.

- a) How much baseboard is needed to surround the room?

$$P = 2(15 \text{ ft}) + 2(12 \text{ ft}) = 30 + 24 = 54 \text{ ft}$$

- b) How much carpet is needed to cover the floor?

$$A = L \cdot W$$

$$= (12 \text{ ft})(15 \text{ ft}) = 180 \text{ ft}^2$$

3.3 Solve Applications - One Unknown

Translating Word Phrases to Algebra

4 important words:

sum
 the difference of x and y
 product
 quotient

<u>Addition</u> the sum of a number and 2 $x+2$ 5 more than a number $x+5$ 3 added to t $t+3$ a number increased by 10 $n+10$	<u>Subtraction</u> the difference of 3 and a number $3-x$ 3 less than y $y-3$ 7 subtracted from a number $n-7$ x decreased by 8 $x-8$
<u>Multiplication</u> the product of 4 and a number $4x$ half of x $\frac{1}{2}x$ twice a number $2x$	<u>Division</u> the quotient of a number and 11 $\frac{n}{11}$ 4 divided by a number $\frac{4}{n}$ \$15 per 5 gallons $\$15/5\text{gal} = \$3/\text{gal}$

Equal

equals
 is equal to
 is

is the same as
 produces
 yields

4 divided into a number $n/4$

results in

Ex a Translate and solve: 4 less than a number is 18

$$n - 4 = 18$$

$$n = 22$$

Ex b Seven more than the product of 8 and a number yields 47. Find the number.

$$8n + 7 = 47$$

$$8n = 40, n = 5$$

Ex c Translate and solve: The sum of n and 3 subtracted from 12 times n equals -11 plus the product of 2 and the difference of n and 5.

$$12n - (n + 3) = -11 + 2(n - 5)$$

$$12n - n - 3 = -11 + 2n - 10$$

$$9n - 3 = -21$$

$$9n = -18, n = -2$$

Applications- construct an equation using a variable for the unknown quantity

Ex d Ben's beginning bank balance was \$258. He deposited \$360, made a withdrawal, and his final balance was \$146. How much did he withdraw?

Ex e The Greens had a 20-lb bag of bird seed. Mice ate some of the seed. The Greens then bought an 8-lb bag of seed and put all the seed in a metal container. They now have 24 lb. How much did the mice eat?

A shipment of 30 crates of cereal boxes was received. If 284 boxes were sold and 76 boxes were left, how many boxes were in each crate?

Practice Problems

1. Four times the sum of a number and 3 results in that number. Find the number.
2. The difference of 31 and the product of 4 and a number equals 3. Find the #.
3. Don weighs 184 lb. His weight is 2 lb less than 6 times his daughter's weight. What is her weight?