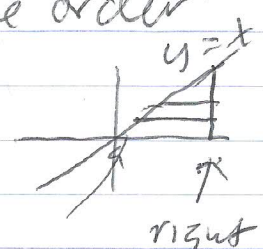
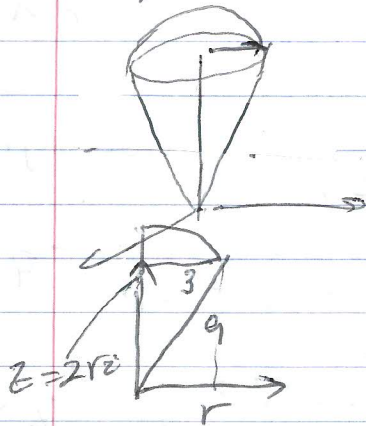


# Math 173 Exam "4" Solutions

1.  $\int_0^1 \int_y^1 \frac{e^x}{x} dx dy$  | Exchange order  
 (type 2)  $= \int_0^1 \int_{y=0}^{y=x} \frac{e^x}{x} dy dx$  |  $x: [y, 1]$   
 $= \int_0^1 \frac{e^x}{x} y \Big|_{y=0}^{y=x} dx$  |  $y: [1, 0]$   
 $= \int_0^1 \frac{e^x}{x} x dx = e^x \Big|_0^1 = e - 1$



2. Volume - 4 methods  $f(x, y, z)$
- 1) Triple integral of 1 - rectangular coord.
  - 2) " " " " - cylindrical "
  - 3) " " " " - spherical "
  - 4) Double integral of  $f(x, y) - g(x, y)$   
 top  $\uparrow$  bottom  $\downarrow$



$y \geq 0 \Rightarrow 0 \leq \theta \leq \pi$   
 $x^2 + y^2 + z^2 = 81$  and  $z = 2r^2 \sqrt{x^2 + y^2}$   
 $\hookrightarrow z^2 = 8(x^2 + y^2)$

Intersection

$$(x^2 + y^2) + 8(x^2 + y^2) = 81$$

$$x^2 + y^2 = r^2 = 9, \quad r = 3$$

$$\sin \phi = \frac{3}{9} = \frac{1}{3}, \quad \phi = \sin^{-1} \frac{1}{3}$$

- 1) Setup Rect:  $\int_{x=-3}^3 \int_{y=0}^{\sqrt{9-x^2}} \int_{z=2r^2 \sqrt{x^2+y^2}}^{\sqrt{81-x^2-y^2}} dz dy dx$
- 2) Setup cyl:  $\int_{\theta=0}^{\pi} \int_{r=0}^3 \int_{z=2r^2 r}^{\sqrt{81-r^2}} r dz dr d\theta$
- 3) Setup sph:  $\int_{\theta=0}^{\pi} \int_{\phi=0}^{\sin^{-1} 1/3} \int_{\rho=0}^9 \rho^2 \sin \phi d\rho d\phi d\theta$
- 4) Set up 2-D cyl:  $\int_0^{\pi} \int_0^3 (\sqrt{81-r^2} - 2r^2 r) r dr d\theta$   
 top (sphere) - bottom (cone)

