Math 173 Exam 2 – Review

Sample Problems – These are meant to be representative of the level of difficulty of what might be on an exam – it's a small sample, not meant to be an all-inclusive list of problem types.

I consider this sample to be "on the long side" for the amount of time we have allotted for a test.

$\vec{T} = \langle \frac{2}{3}(1+t)^{1/2}, -\frac{2}{3}(1-t)^{1/2}, \frac{1}{3} \rangle$

1. Find T(t), N(t), and B(0) for the curve $\langle \frac{4}{9}(1+t)^{\frac{3}{2}}, \frac{4}{9}(1-t)^{\frac{3}{2}}, \frac{1}{3}t \rangle$ $\vec{N} = \frac{1}{\sqrt{2}} \langle (1-t)^{1/2}, (1+t)^{1/2}, 0 \rangle$

   $\vec{B}(0) = \langle -\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}} \rangle$

2. a) Prove that the following limit exists: $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$ Squeeze Thm → $\frac{x^2}{x^2+y^2} \le 1$

   b) Prove that the following limit does not exist: $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ test curves $y=0$, $y=x^2$

3. a) Find the equation of the tangent plane of $f(x,y) = \frac{4x}{y}$ at the point (1, 2, 2) $-2x+y+z=2$

   b) Write the linearization $L(x,y)$ of f $z = L(x,y) = +2x - y + 2$

   c) Use the linearization of f to approximate f at the point (1.1, 2.1) ≈ 2.1

4. Find $\frac{dy}{dx}$ for $y^2 - x^2 - \sin(xy) = 0$ using partial derivates f_x, f_y $\frac{2x + y\cos xy}{2y - x\cos xy}$

5. Find the directional derivative of $f(x,y) = x^2y + e^{xy}\sin y$ at the point (1, 0) in the direction of the point (3, -3) $-6/\sqrt{13}$

6. Find the maximum rate of change of $f(x,y) = x^2y + e^{xy}\sin y$ at the point (1, 0), and the direction in which it occurs. max is $|\langle 0,2 \rangle| = 2$, direction is $\langle 0,2 \rangle$

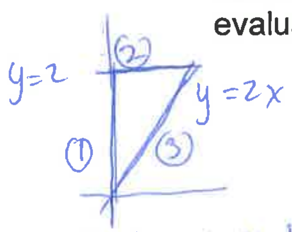
7. Use the chain rule to find $\frac{\partial w}{\partial r}$ *and* $\frac{\partial w}{\partial s}$ when r = 1 and s = -1 for

   $w = (x+y+z)^2$ $\partial w/\partial r = 12$, $\partial w/\partial s = 0$
   $x = r - s$
   $y = \cos(r+s)$
   $z = \sin(r+s)$

8. a) Find any local maximum or local minimum values of

   $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ local min at (1, 2) $f(1,2) = -5$

   b) Find the absolute maximum and minimum values of the triangular region D, bounded by the lines x = 0, y = 2, and y = 2x in the first quadrant. Draw a sketch of D, list all critical points (including interior points in part "a"), and evaluate the function f at those critical points.



(1) $x=0$: $f(0,y) = y^2 - 4y + 1$ $f_y = 2y - 4 = 0$ at $y = 2$
only $y \in [0,2]$ $f(0,0) = 1$

(2) $y = 2$; $f(x,2) = 2x^2 - 4x - 3$, $f_x = 4x - 4 = 0$, $x = 1$
already tested

(3) $y = 2x$; $f(x,2x) = 6x^2 - 12x + 1$, $f_x = 12x - 12 = 0$
$x = 1$, already tested

x	y	f(x,y)
1	2	-5
0	2	-3
0	0	1

$f(1,2) = -5$ abs min
$f(0,0) = 1$ abs. max