Math 173 Exam 4 Name Key

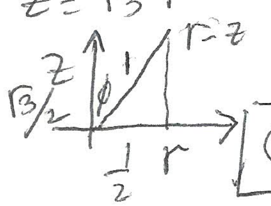
Problem 1: 12 pts, Problems 2- 6: 14 points. Problem 7: 18 points.

Bonus problems are on the back page and are extensions of the "main" problems.

1. DO NOT INTEGRATE. Convert to spherical coordinates, showing correct limits of integration:

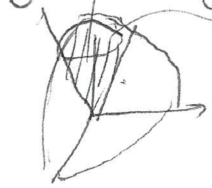
$\iiint_E \overbrace{\sin((x^2+y^2+z^2)^{3/2})}^{\sin \rho^3}\, dV$, where E is the solid bounded above by the sphere $\underbrace{x^2+y^2+z^2}_{}=1$ and below by the cone $z=\sqrt{\underbrace{3x^2+3y^2}_{3r^2}}$

$z = \sqrt{3}\, r$



$\boxed{\phi = \pi/6}$

$\rho^2 = 1 \Rightarrow \rho = 1$



Volume, not surface

$\rho : [0, 1]$

$\phi : [0, \pi/6]$

$\theta : [0, 2\pi]$

$$\int_0^{2\pi}\int_0^{\pi/6}\int_0^1 \underbrace{\rho^2 \sin\phi}_{\text{conversion factor}} \cdot \overbrace{\sin \rho^3}^{\text{function}}\, \underbrace{d\rho\, d\phi\, d\theta}_{dV}$$

$u = \rho^3 \quad du = 3\rho^2 d\rho$

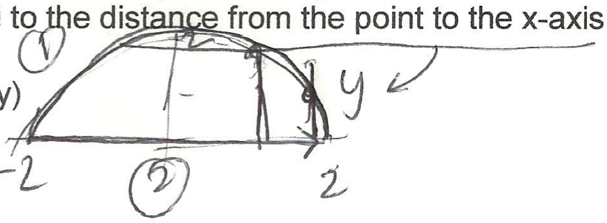
Bonus $\int_0^{2\pi} d\theta \int_0^{\pi/6} \sin\phi\, d\phi \int_0^1 \rho^2 \sin \rho^3\, d\rho$

$= \theta\Big]_0^{2\pi} \cdot -\cos\phi\Big]_0^{\pi/6} \cdot -\frac{1}{3}\cos\rho^3\Big]_0^1$

$= 2\pi\left[\frac{\sqrt{3}}{2} - 1\right]\cdot\left[\frac{\cos 1 - 1}{3}\right] = \frac{2\pi}{3}\underbrace{\left[\frac{\sqrt{3}}{2}-1\right]\cdot\left[\cos 1 - 1\right]}_{4\ \text{terms}}$

$= \frac{\cancel{2}\pi}{\cancel{2}\cdot 3}(\sqrt{3}-2)(\cos 1 - 1) = \frac{\pi}{3}(\sqrt{3}\cos 1 - \sqrt{3} - 2\cos 1 + 2)$

2. A thin wire is bent into the shape of the semi-circle $x^2 + y^2 = 4$ for $y \geq 0$. The linear density of the wire at any point is proportional to the distance from the point to the x-axis.



a) Write a function for the linear density, ρ(x,y)

$\rho(x, y) = ky$

b) Find the mass of the wire.

Line integral, not area. There are 2 parts to wire, but straight part has $\rho = 0 \Rightarrow m = 0$

Use $x = r\cos t = 2\cos t$ $dx/dt = -2\sin t$

$y = r\sin t = 2\sin t$ $dy/dt = 2\cos t$

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $t: [0, \pi]$ for $y \geq 0$

$\int \rho \, ds$

$= \int_0^{\pi} k \underbrace{y}_{2\sin t} \underbrace{\sqrt{4\sin^2 t + 4\cos^2 t}}_{=2}\, dt + \int_{-2}^{2} k(0)\, dx$

$= \int_0^{\pi} 2k \sin t \cdot 2\, dt + 0\Big]_{-2}^{2}$

$= 4k(-\cos t)\Big]_0^{\pi}$

$= -4k[-1 - 1] = 8k$