3 and 4 should not be closed integrals

Math 173 Exam 4 Review

All curves and surfaces are oriented in the positive direction. Give exact answers (no decimal approximations). If a specific method is stated, use it; if not stated, you may use the method of your choice.

ρ $\quad (x,y,z) \to (\rho,\theta,\phi)$

1. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$ using spherical coordinates.

$z: [\sqrt{x^2+y^2}, \sqrt{2-x^2-y^2}] \Rightarrow z=r$ to $z=\sqrt{2-r^2}$

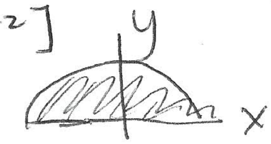
$\phi \le \pi/4$ limit

$z^2 = 2-r^2;\ z^2+r^2=2$

$\rho^2 = 2$

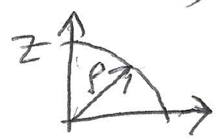
$\rho = \sqrt{2}$ limit

$y: [0, \sqrt{1-x^2}]$



$x: [-1, 1]$

$\Rightarrow 0 \le \theta \le \pi$



$\int_0^{\pi/4}\int_0^{\pi}\int_0^{\sqrt{2}} \rho\cdot\rho^2\sin\phi\,d\rho\,d\theta\,d\phi = \frac{\rho^4}{4}\Big]_0^{\sqrt{2}}\cdot\theta\Big]_0^{\pi}\cdot -\cos\theta\Big]_0^{\pi/4} = 1\cdot\pi\left(1-\frac{\sqrt{2}}{2}\right)$

2. Set up the integral for transformation, but do not intgrate: $\iint_R x^3 y\,dA$. where R is R is the region bounded by $xy=1$, $xy=3$, $y=2$, $y=6$ using $x=\frac{v}{6u}$, $y=2u$

Jacobian: $\begin{vmatrix} \frac{-v}{6u^2} & \frac{1}{6u} \\ 2 & 0 \end{vmatrix} = \left|0-\left(\frac{1}{6u}\right)(2)\right| = \left|-\frac{1}{3u}\right|$

$y: [2,6]$; $u=\frac{y}{2} \Rightarrow u: [1,3]$

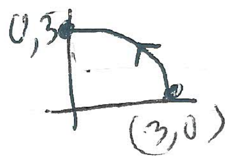
$x = v/6u \Rightarrow x = v/3y$; $v = 3xy$; $xy: [1,3] \Rightarrow v: [3,9]$

$\iint_R x^3 y\,dA = \int_1^3\int_3^9 \left(\frac{v}{6u}\right)^3\cdot(2u)\left|\frac{-1}{3u}\right|\,dv\,du$ ← u positive

Integral should not be closed

$= \int_1^3\int_3^9 \frac{8u^3 v}{6u}\cdot\frac{1}{3u}\,dv\,du = \int_1^3\int_3^9 \frac{4}{9}uv\,dv\,du$

3. $\oint_C f\,dS$ where $f(x,y)=y$ and C is the quarter circle $x^2+y^2=9$ from (3, 0) to (0, 3) ← $t=0$ ← $t=\pi/2$



$x = 3\cos t \quad dx/dt = -3\sin t \quad f(x,y)=y$

$y = 3\sin t \quad dy/dt = 3\cos t \quad f_x = 0,\ f_y = 1$

$\int f\,ds = \int_0^{\pi/2} f(x,y)\sqrt{(dx/dt)^2+(dy/dt)^2}\,dt$

$= \int_0^{\pi/2} 3\sin t\sqrt{9\sin^2 t+9\cos^2 t}\,dt = 9\int_0^{\pi/2}\sin t\,dt$

$= -9\cos t\Big]_0^{\pi/2} = -9(0-1) = 9$