Math 173 Exam 4 Traditional Problems   Name ______________________

Directions:

- Write your solutions on separate paper with problem numbers clearly marked and pages/problems in numerical order.
- To earn full points, follow the instructions for each problem. Most problems require multiple set-ups for solution, and some have restrictions/conditions.
- Upload copies of your solutions to the Canvas Exam 4 area before the deadline.
- Paper notes and a calculator may be used; NO HELP FROM PEOPLE OR ONLINE SOURCES PERMITTED.
- When asked to set up an integral, show all limits of integration and variables of integration for full credit.

Point values for each problem are marked.

1. (6 pts) Draw the region of integration and evaluate: $\int_0^1 \int_y^1 \frac{e^x}{x}\, dx\, dy$

2. (12 pts) The problem: Find the volume of the solid where $y \geq 0$, bounded (above) by the sphere $x^2 + y^2 + z^2 = 81$, and bounded below by the cone, $z = 2\sqrt{2}\sqrt{x^2 + y^2}$.

   a) The set-up: Set up integrals to solve the problem in **4** unique ways, using methods from sections 12.1 – 16.9 (anything from whole course). Exchanging the order of integration will NOT be counted as a “unique” set-up for this problem.

   b) The solution: Choose one of your methods and solve the problem. Explain why you chose that method.

3. (10 pts) For the integral $\iint_D \ (x + y) \cdot e^{x-y}\ \ dx\, dy$ where region D is the triangle with vertices at (0,0), (-1,1) and (1,1)
   a) Create a transformation using variables $u\ and\ v$ for integration, and set up an integral using the transformed variables.
   b) Show graphs of the original area, D, and also the transformed area.
   c) Perform the integral, either using the original or the transformed integral.

4. (10 pts) The problem: The temperature on a shop floor (with heating equipment) is given as $f(x, y) = 4x^2 - 4xy + y^2$. The region contains a circular barrier with radius = 5, centered at the origin. Find the maximum and minimum temperatures along the barrier (edge of the circle).
   a) Identify the function to be maximized/minimized, and the constraint equation(s).

   b) Find the maximum and minimum temperatures along the barrier.

5. (10 pts.) A volume is calculated as: $\int_0^4 \int_0^{\sqrt{4-\frac{y^2}{4}}} \int_0^{4-y} dx\, dz\, dy$.

   Note: $\sqrt{4-\frac{y^2}{4}}$ can also be represented as $\frac{\sqrt{16-y^2}}{2}$

   a) Use the limits of integration to draw the shape of the volume OR write equations describe the boundaries and describe the region.

   b) Find the limits of integration for the order $\iiint \quad dz\, dx\, dy$

   c) Find the limits of integration for the order $\iiint \quad dy\, dz\, dx$

6. (10 pts) A thin plate covers the triangular region defined by the vertices (0,0), (1,0), and (1, 2). The plane's density at the point (x, y) is $\rho(x, y) = x + y + 1$

   a) Find the second moments of inertia, I_x *and* I_y

   b) Why do you think I_x is greater than I_y ? Give a descriptive answer based on the physical properties of the plate.

7. (12 pts) The problem: For $\boldsymbol{F} = \langle x + 2xy, x^2 \rangle$ find the integral $\int_C \boldsymbol{F} \cdot \boldsymbol{dr}$ where C consists of the line segments first from (0,0) to (2,1), then from (2,1) to (3, 0).

   a) The set-up: Set up integrals to solve the problem in **3** unique ways, using methods from sections 12.1 – 16.9 in this course.

   b) Choose one method and solve the problem completely. Explain why you chose that method, and what conditions must be met to use that method.

   c) Suppose the line segment from (3, 0) to (0,0) is added to the path above, so C is composed of 3 line segments, including the previous 2 segments. Now, what is the line integral $\int_C \boldsymbol{F} \cdot \boldsymbol{dr}$? Show your work or explain your answer.

8. (12 pts) For the integral: $\oint_C xy\, dx + (x + y)\, dy$ where C is the counterclockwise path traced on the unit disk, $x^2 + y^2 \leq 1$.

   a) Set up integrals by 2 different methods.

   b) Evaluate the integral using one of the methods.

   c) List all conditions required for the 2 methods to be used interchangeably

   d) Describe a physical scenario where the above integration might need to be computed. Assign physical properties to each of the associated quantities.

9. (8 pts) For the problem, "Find the surface area of the band cut from sphere $x^2 + y^2 + z^2 = 4$, bounded by the planes $z = 1 \; and \; z = \sqrt{3}$."
   a) Write a parameterization for the surface. Explain why you chose the parameters you used.
   b) Set up the integral (including limits of integration) to calculate the surface area. If you define the vectors $\boldsymbol{r}, \boldsymbol{r_u}, \boldsymbol{and}\ \boldsymbol{r_v}$, you may use the expression $|\boldsymbol{r}_u \times \boldsymbol{r}_v|$ in your set-up without calculating $|\boldsymbol{r}_u \times \boldsymbol{r}_v|$ and without actually integrating.
   c) (bonus: 2 points) Calculate $|\boldsymbol{r}_u \times \boldsymbol{r}_v|$, and complete the integral to find the surface area.

10. (10 pts) a) Find curl $\boldsymbol{F} = \nabla\, X\, \boldsymbol{F}$ for $\boldsymbol{F} = \langle x^2y, xyz, -x^2y^2 \rangle$
    b) Is F conservative?
    c) Find div (curl $\boldsymbol{F}$) (the divergence of the curl of $\boldsymbol{F}$)