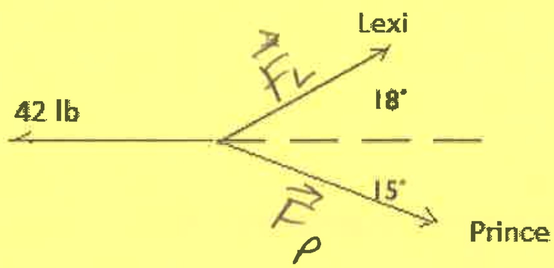
Math 173 Exam 1 Name Key

Directions:

- Show all work on this test to receive partial credit – scratch work will not be examined for credit.
- Leave answers in exact form, unless otherwise specified. Radicals in the denominator will not be penalized.

Problems #1 – 6 are each 14 points. Problems #7 and 8 have points as marked.

1. Two dogs (Lexi and Prince) are taken for a walk. Stopping them at a curb requires 42 lb of force. Lexi pulls at an angle of 18° from the walking direction, and Prince pulls at an angle of 15°. What is the force exerted on each leash (to the nearest pound)?



Vertical: $|\vec{F}_L| \sin 18 = |F_P| \sin 15°$

$|\vec{F}_P| = |\vec{F}_L| \dfrac{\sin 18°}{\sin 15°} \approx \dfrac{.3090}{.2588} |F_L| \approx 1.194 F_L$

(you could also have solved for $|\vec{F}_L|$

Horizontal:

$42 = |\vec{F}_L| \cos 18° + |\vec{F}_P| \cos 15°$

$42 \approx .951 |\vec{F}_L| + (1.194)\cdot(.966) F_L \approx 2.100 F_L$

$|F_L| = \dfrac{42}{2.1} = 20 \text{ lb}$ (at 18°)

$|F_p| = 1.194 (20) \approx 24 \text{ lb}$ (at −15°)

2. a) Find the cosine of the angle between the planes $y + z = 1$ and $2x + y - z = 0$. Give exact answers (not decimal approximations).

$\vec{n_1} = \langle 0, 1, 1 \rangle$ $\quad \vec{n}_2 = \langle 2, 1, -1 \rangle$

$\vec{n}_1 \cdot \vec{n}_2 = |n_1||n_2| \cos\theta$

$$\cos\theta = \frac{\langle 0,1,1\rangle \cdot \langle 2,1,-1\rangle}{\sqrt{0+1+1} \cdot \sqrt{4+1+1}} = \frac{0+1-1}{\sqrt{2}\sqrt{6}} = \frac{0}{\sqrt{12}}$$

$= 0$

b) Represent the line of intersection between the 2 planes in parametric form.

need ① parallel vector and ② point

① a vector orth. to n_1 and orth. to n_2 is parallel to both

$$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$\frac{n_1 \times n_2}{2}$ is still parallel $\Rightarrow$ let $v = \langle -1, 1, -1 \rangle$

② A point on line satisfies both planes

Let $y = 0$

$0 + z = 1 \Rightarrow z = 1$

$2x - z = 0 \Rightarrow x = 1/2$

$(1/2, 0, 1)$

One of many possible points

Equations (Parametric)

$x = 1/2 - t$

$y = t$

$z = 1 - t$

3. a) Find parametric and symmetric representations of the line passing through the points P(3, -1, 2) and Q(-2, 0, 2)

$\overrightarrow{PQ} = \langle -5, 1, 0 \rangle$ = direction vector

Use P (or Q) as point

$t = \dfrac{x-3}{-5} = y+1 \; ; \; z = 2$

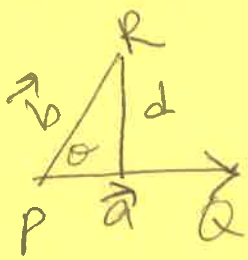
$x = 3 - 5t$

$y = -1 + t$

$z = 2$

call it R

b) Find the orthogonal distance from the point (0, 5, 4) to the line above.



$d = |\vec{b}| \sin\theta = \dfrac{|a \times b|}{|a|}$

Use cross product because we want $|b| \sin\theta$. This occurs because we want distance to a line, not a plane

Let $\vec{a} = \overrightarrow{PQ}$

$b = \overrightarrow{PR} = \langle -3, 6, 2 \rangle$

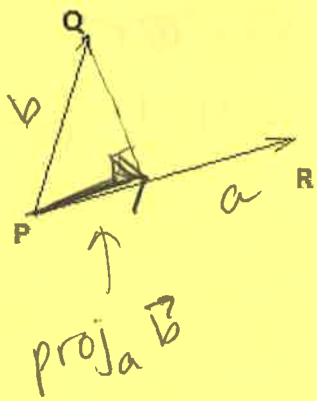
$$|a \times b| = \left\| \begin{matrix} i & j & k \\ -5 & 1 & 0 \\ -3 & 6 & 2 \end{matrix} \right\| = |2i - 10j - 27k|$$

$$= \sqrt{4 + 100 + 729} = \sqrt{833}$$

$$d = \frac{|a \times b|}{|a|} = \frac{\sqrt{833}}{\sqrt{26}} = \frac{7\sqrt{17}}{\sqrt{26}}$$

good enough

4. The coordinates of 3 points are P(2, 0, 1), Q(0,0,4) and R(-1,1,0).



a) Let $\mathbf{a} = \overrightarrow{PR}$ and $\mathbf{b} = \overrightarrow{PQ}$.
Draw $\text{proj}_\mathbf{a}\, \mathbf{b}$ on the diagram and find the vector projection

$$\text{proj}_a \vec{b} = \frac{a \cdot b}{|a|} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\vec{a} = \langle -3, 1, -1 \rangle \quad \vec{b} = \langle -2, 0, 3 \rangle$$

$$a \cdot b = 6 + 0 - 3 = 3$$

$$\text{proj}_a \vec{b} = \frac{3}{\sqrt{11}} \cdot \frac{\langle -3, 1, -1 \rangle}{\sqrt{11}} = \frac{3}{11} \langle -3, 1, -1 \rangle$$

$$= \langle -9/11, 3/11, -3/11 \rangle$$

b) Find the area of the triangle defined by P, Q, and R

Area of parallelogram $= |a \times b|$; arlea of $\triangle = \frac{1}{2}|a \times b|$

$$|a \times b| = \begin{vmatrix} i & j & k \\ -3 & 1 & -1 \\ -2 & 0 & 3 \end{vmatrix} = |3i + 11j + 2k|$$

$$= \sqrt{9 + 121 + 4} = \sqrt{134}$$

Area of $\triangle = \frac{1}{2}\sqrt{134}$ (units^2)

5. a) Write the quadric surface equation in standard form $4x^2 - y^2 + z^2 + 8x = 0$

$$4x^2+8x+4 \quad -y^2+z^2=0+4$$

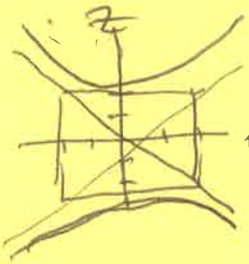
$$\frac{4(x+1)^2}{4}-\frac{y^2}{4}+\frac{z^2}{4}=\frac{4}{4}$$

$$\frac{(x+1)^2}{1}-\frac{y^2}{4}+\frac{z^2}{4}=1$$

b) Draw 3 traces, for x = -1, y = 0, and z = 0

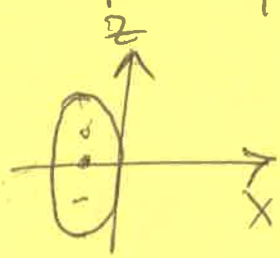
$x=-1$ hyperbola

$$\frac{z^2}{4}-\frac{y^2}{4}=1$$



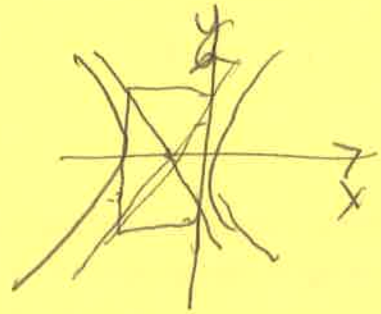
$y=0$ ellipse

$$\frac{(x+1)^2}{1}+\frac{z^2}{4}=1$$

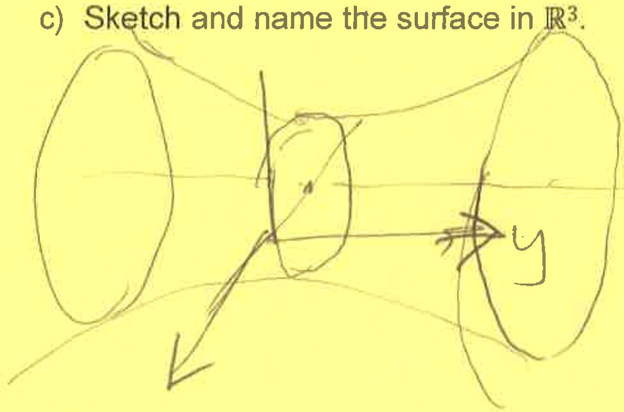


$z=0$

$$\frac{(x+1)^2}{1}-\frac{y^2}{4}=1$$



c) Sketch and name the surface in $\mathbb{R}^3$. Hyperboloid of 2 sheets

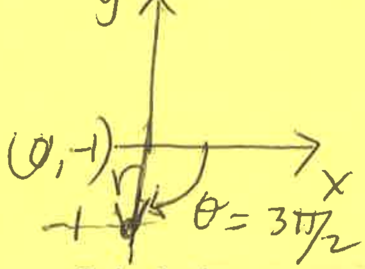


6. a) Convert the point (0, -1, $\sqrt{3}$) from rectangular (Cartesian) coordinates to cylindrical coordinates and spherical coordinates

can see by drawing

Cylindrical coordinates

to get r, θ



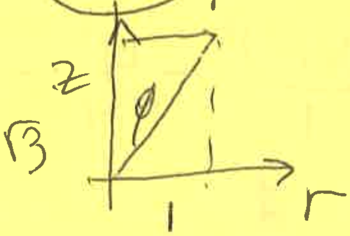
$r = \sqrt{0^2 + (-1)^2} = 1$ ← need drawing

$\theta = 3\pi/2$

$z = \sqrt{3}$ (same)

Spherical coordinates

r - z plane



$\rho = \sqrt{1^2 + \sqrt{3}^2} = 2$

$\theta = 3\pi/2$ (same as above)

$\phi = \tan^{-1}\frac{1}{\sqrt{3}} = \pi/6$

$(1, 3\pi/2, \pi/6)$

b) Write inequalities for each coordinate in cylindrical coordinates to define the region that lies outside the cylinder $x^2 + y^2 = 1$ and inside the hemisphere $z = \sqrt{9 - x^2 - y^2}$

top z is $\sqrt{9 - x^2 - y^2}$ pos. z only

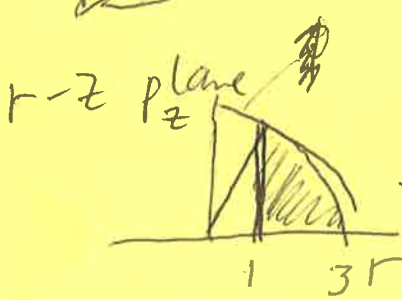
$= \sqrt{9 - r^2}$

$1 \le r \le 3$

$0 \le z \le \sqrt{9 - r^2}$

$0 \le \theta \le 2\pi$

r - z plane



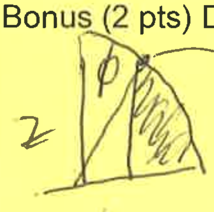
Cylindrical

$\csc\phi \le \rho \le 3$

$\sin^{-1}\frac{1}{3} \le \phi \le \pi/2$

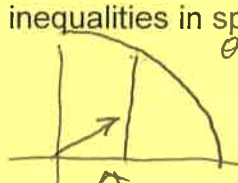
$0 \le \theta \le 2\pi$

Bonus (2 pts) Define the region above using inequalities in spherical coordinates.



begins at $\phi = \sin^{-1}\frac{1}{3}$

outside $\rho = 3$



inside

$\rho = \frac{(r) = 1}{\sin\phi} = \csc\phi$

7. (12 pts.) For the curve $r(t) = \langle \ln(t-3), \sqrt{t}, -\frac{t^2}{2} \rangle$

a) Give the domain of the curve

$x: (3, \infty)$
$y: [0, \infty)$
$z: (-\infty, \infty)$

Domain: $(3, \infty)$

b) Find the equation line tangent to the curve at (0, 2, - 8), writing your final answer in parametric form.

$r'(t) = \langle \frac{1}{t-3}, \frac{1}{2}t^{-1/2}, -t \rangle$

$r'(4) = \langle 1, 1/4, -4 \rangle \leftarrow t=4 \Leftarrow$

$\ln(t-3) = 0$
$\sqrt{t} = 2$
$-t^2/2 = -8$

$x = 0 + t$

$y = 2 + \frac{1}{4}t$

$z = -8 - 4t$

8. (4 pts.) Demonstrate that the vectors $\langle 0, 3, -6 \rangle$ and $\langle 5, 4, 2 \rangle$ are orthogonal.

$\vec{a}$ $\vec{b}$

$\vec{a} \cdot \vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are orthogonal

$\vec{a} \cdot \vec{b} = 0 \cdot 5 + 3 \cdot 4 - 6 \cdot 2 = 0 + 12 - 12 = 0$