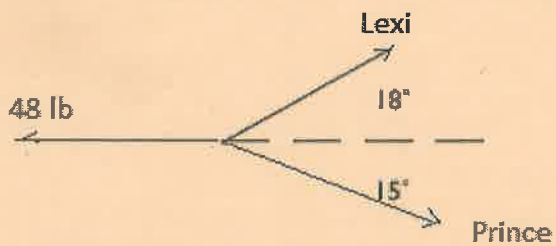
Math 173 Exam 1 Name

Problems #1 – 6 are each 14 points. Problems #7 and 8 have points as marked.

1. Two dogs (Lexi and Prince) are taken for a walk. Stopping them at a curb requires 48 lb of force. Lexi pulls at an angle of 18° from the walking direction, and Prince pulls at an angle of 15°. What is the force exerted on each leash (to the nearest pound)?



Vert: $|F_L| \sin 18° = |\vec{F}_P| \sin 15°$

$$|\vec{F}_P| = |\vec{F}_L| \frac{\sin 18°}{\sin 15°} \approx \frac{.3090}{.2588} |F_L|$$

$$\approx 1.194 |F_L|$$

Horizontal:

$$48 = |\vec{F}_L| \cos 18° + |\vec{F}_P| \cos 15°$$

$$48 = |F_L|(0.951) + 1.194|F_L|(.966)$$

$$= 2.100 |F_L|$$

$|F_L| = \frac{48}{2.1} \approx 23$ lb (at 18°)

$|F_P| = 1.194(22.86) \approx 27$ lb (at 15°)

more digits

2. a) Find the cosine of the angle between the planes $x + y = 1$ and $2x + y - z = 0$. Give exact answers (not decimal approximations).

b) Represent the line of intersection between the 2 planes in parametric form.

3. a) Find parametric and symmetric representations of the line passing through the points P(3, -1, 1) and Q(-2, 0, 1)

$\vec{v} = \langle -5, 1, 0 \rangle$ $\qquad t = \frac{x+2}{-5} = y \;;\; z = 1$

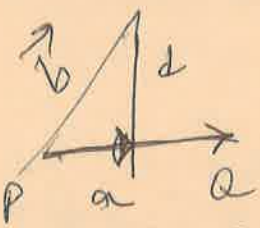
$x = -2 - 5t$

$y = 0 + t$

$z = 1$

call it R

b) Find the orthogonal distance from the point (0, 5, 2) to the line above.



$d = |\vec{b}| \sin\theta = \frac{|\vec{a \times b}|}{\vec{a}}$

use cross product - want $\sin\theta$

Want dist. to line, not plane

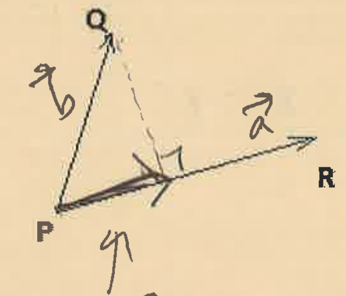
Let $\vec{a} = \vec{PQ}$

$b = \vec{PR} = \langle -3, 6, 1 \rangle$

$|a \times b| = \left\| \begin{matrix} i & j & k \\ -5 & 1 & 0 \\ -3 & 6 & 1 \end{matrix} \right\| = |1\, i + 5 j - 27 k| = \sqrt{755}$

$\frac{|a \times b|}{|a|} = \frac{\sqrt{755}}{\sqrt{26}}$

4. The coordinates of 3 points are P(2, 0, 1), Q(0,0,3) and R(-1,1,0).



a) Let $\mathbf{a} = \overrightarrow{PR}$ and $\mathbf{b} = \overrightarrow{PQ}$.
Draw $\text{proj}_\text{a}\, \mathbf{b}$ on the diagram and find the vector projection.

$\vec{a} = \langle -3, 1, -1 \rangle$ $\quad a \cdot b = 6 + 0 - 2 = 4$

$\vec{b} = \langle -2, 0, 2 \rangle$

$$\text{proj}_a \vec{b} = \frac{a \cdot b}{|a|} \cdot \frac{\vec{a}}{|a|} = \frac{4}{\sqrt{11}} \frac{\langle -3, 1, -1 \rangle}{\sqrt{11}} = \frac{4}{11} \langle -3, 1, -1 \rangle$$

b) Find the area of the triangle defined by P, Q, and R

$$A = \frac{1}{2}|a \times b| = \frac{1}{2}\left\| \begin{matrix} i & j & k \\ -3 & 1 & -1 \\ -2 & 0 & 2 \end{matrix} \right\| = \frac{1}{2}\left| 2\hat{i} + 8\hat{j} + 2\hat{k} \right|$$

$$= \frac{1}{2}\sqrt{4 + 64 + 4} = \frac{1}{2}\sqrt{72} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

5. a) Write the quadric surface equation in standard form $x^2 - y^2 + 4z^2 + 8z = 0$

$$X^2 - y^2 + 4(z^2 + 2z + 1) = 0 + 4$$

$$\frac{X^2}{4} - \frac{y^2}{4} + \frac{\cancel{4}(z+1)^2}{\cancel{4}} = 1$$

b) Draw 3 traces, for x = 0, y = 0, and z = -1

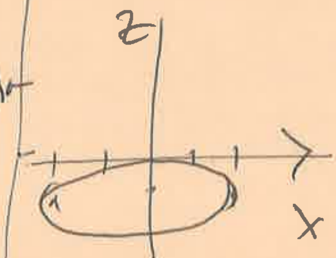
$X = 0$: $\frac{(z+1)^2}{1} - \frac{y^2}{4} = 1$

Vert. Hyperbola

$y = 0$

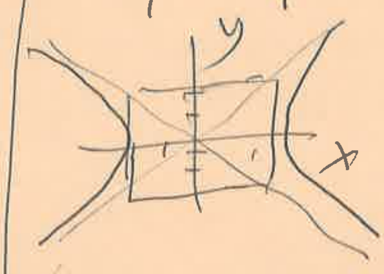
$\frac{X^2}{4} + \frac{(z+1)^2}{1} = 1$

wider than tall



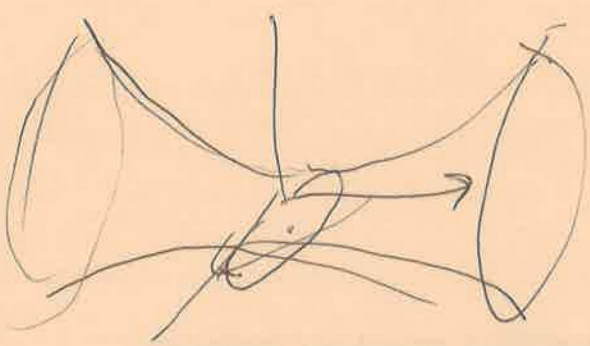
$z = -1$

$\frac{X^2}{4} - \frac{y^2}{4} = 1$



horiz. hyperbola

c) Sketch and name the surface in $\mathbb{R}^3$.



See yellow

6. a) Convert the point (0, -1, $\sqrt{3}$) from rectangular (Cartesian) coordinates to cylindrical coordinates and spherical coordinates

Cylindrical coordinates

Spherical coordinates

b) Write inequalities for each coordinate in cylindrical coordinates to define the region that lies outside the cylinder $x^2 + y^2 = 1$ and inside the hemisphere $z = \sqrt{9 - x^2 - y^2}$

Bonus (2 pts) Define the region above using inequalities in spherical coordinates.

see yellow

7. (12 pts.) For the curve $r(t) = \langle \ln(t-3), \sqrt{t}, -\frac{t^2}{2} \rangle$
   a) Give the domain of the curve

   b) Find the equation line tangent to the curve at $(0, 2, -8)$, writing your final answer in parametric form.

8. (4 pts.) Demonstrate that the vectors $\langle 2, 3, 0 \rangle$ and $\langle 6, -4, 2 \rangle$ are orthogonal.