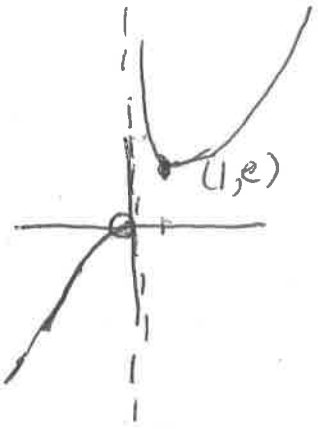
Math 171 – Exam 3 Review

Sample Problems – These are the problems from an old exam – it's a small sample, not meant to be an all-inclusive list of problem types

1. Find the absolute maximum and the absolute minimum values of the function $f(x) = -\frac{1}{x^2}$ on the interval [0.5, 2].
2. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for the function: $f(x) = x^{2/3}$ on the interval [0, 1]
3. Use L'Hospital's Rule to find $\lim_{x\to 0} \frac{3x^2}{\cos x - 1}$
4. All units in a 100 unit apartment building are rented out when the monthly rent is set at $r = \$900 / \text{month}$. Suppose that one unit becomes vacant with each \$10 increase in rent and that each occupied unit costs \$80 / month in maintenance. Which rent r maximizes monthly profit?
5. Use Newton's method to estimate the two zeros of the function $f(x) = x^4 + x - 3$. Start with $x_1 = 1$ for the zero on the right (higher value of x). Then in each case, find x_3.
6. a) If $f''(x) = \frac{1}{x^2} + 6x$, find the general form of $f'(x)$
   b) Find the general form of $f(x)$
   c) Find $f(x)$ when $f'(1) = 4$ and $f(1) = 0$
7. For $f(x) = xe^{1/x}$
   a) Find the equations of any asymptotes
   b) Give the values of any local maxima or minima
   c) Give the values of any inflection points
   d) Sketch the graph

7d)



Answers:

1. Amax = $-1/4$
   Amin = -4
2. $c = 8/27$
3. -16
5. For $x_1 = -1$
   $x_3 = -1.6452$
   For $x_1 = 1$
   $x_3 = 1.16542$
4. $x = 9$, rent = \$90

6. a) $f'(x) = -\frac{1}{x} + 3x^2 + c$
   b) $f(x) = -\ln x + x^3 + cx + d$
   c) $f(x) = -\ln x + x^3 + 2x - 3$
7. a) VA: $x = 0$ (since $\frac{1}{x}$ undefined at $x = 0$)
   HA: none (since as $x \to \pm\infty$ $f(x) \to \pm\infty \cdot 1$)
   b) Lmin: (1, e)
   c) none (at $x = 0$, concavity changes from C down to C up, but $x = 0$ gives an undefined point)
   d) see above

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Topics covered:

- Finding local and absolute maxima and minima from a graph.
- Finding critical values of an equation
- Finding local and absolute maxima and minima from an equation on a specified interval
- Rolle's Theorem
  - What are the conditions? The conclusion?
  - Finding a value "c" that satisfies Rolle's Theorem
  - Using it in proofs (for example, proving a function has exactly one zero).
- Mean Value Theorem
  - What are the conditions? The conclusion?
- First derivative: regions of increase/decrease, critical values, local maxima and minima.
- Second derivative: regions of concave up, concave down, inflection points
- Be able to give examples of critical values that do not produce local maxima, local minima, or inflection points.
- L'Hospital's Rule – finding the limits of Indeterminate Forms
- Curve sketching (graphing), including finding
  - Domain
  - Intercepts
  - Symmetries
  - Asymptotes
  - Regions of increase/decrease
  - Local maxima and minima
  - Concavity and inflection points
  - Combining simpler functions
  - Strategic points
- Optimization Problems (maxima and minima)
- Finding roots using Newton's Method
- Derivatives of functions and finding antiderivatives, including
  - Polynomials
  - Trig functions
  - Inverse trig functions
  - Log functions (base e and other)
- Displacement, velocity and acceleration equations - using antiderivates