Math 171 – Exam 2 Review

Sample Problems – These are the problems from an old exam – it's a small sample, not meant to be an all-inclusive list of problem types

1. a) Find y' *if* $y = (x^2 + 7x + 2)^{\frac{1}{3}}$ $\quad y' = \frac{(2x+7)}{3(x^2+7x+2)^{2/3}}$

   b) Find y' *if* $y = e^x \tan x$ $\quad y' = e^x(\tan x + \sec^2 x)$

2. Find the equation of the tangent line to the curve for $(2/3, \sqrt{3}/3)$

   $y = \dfrac{\sin x}{1+\cos x}$ when $x = \dfrac{\pi}{3}$ $\quad y' = \frac{\cos x + \sin^2 x + \cos^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{1+\cos^2 x} \cdot \frac{1}{1+\cos x} = \frac{2}{3}$

3. a) Write the linearization of the equation in problem 2. $\quad L(x) = \frac{2}{3}x - \frac{2\pi}{9} + \sqrt{3}/3$

   b) Write the differential, dy, of the equation in problem 2 $\quad dy = \frac{1}{\cos x}dx$

4. Choose and prove ONE of the following statements:

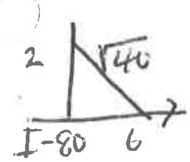
   a) $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$ OR

   b) $\dfrac{d}{dx}\left[b^x\right] = \ln b \cdot b^x \quad$ *for* $b \neq 1,\ b > 0$

   c) $\dfrac{d}{dx}\left[\tan^{-1} x\right] = \dfrac{1}{1+x^2}$ $\quad y = \tan x;\ x = \tan y;\ 1 = \sec^2 y \cdot dy/dx;\ y' = \frac{1}{1+\frac{1}{\cos^2 y}} = \frac{1}{1+x^2}$

5. A road, perpendicular to I-80, leads to a farmhouse located 2 miles away. A car, traveling 80 mph on I-80 passes the exit for the farmhouse. How fast is the distance between the farmhouse and the car~~e~~ increasing when the car is 6 miles past the intersection (section) of I-80 and the road? $\quad 2^2 + x^2 = r^2 \quad 2x \cdot dx/dt = 2r \cdot dr/dt$

   $dr/dt = \frac{6\cdot 80}{\sqrt{40}} = 75.9$ mph



6. If $g(x) = \dfrac{5x^3\sqrt{4x+7}}{e^x(1+x)^2}$ find $g'(x)$. $= \left[\frac{3}{x} + \frac{1}{2(4x+7)} - 1 - \frac{2}{1+x}\right] \cdot \frac{5x^3\sqrt{4x+7}}{e^x(1+x)^2}$

7. For the equation $y^3 + 3x^4 = xy$ find dy/dx

8. The half-life of a radioactive isotope is 80 years. If 20% of the original sample remains, how old is the sample?

7. $3y^2 \cdot dy/dx + 12x^3 = x \cdot dy/dx + y$

$$\frac{dy/dx\,(3y^2 - x)}{3y^2 - x} = \frac{y - 12x^3}{3y^2 - x}$$

8. $\frac{1}{2} = e^{k(80)} \quad \ln .5 = k \cdot 80$

$k = -.693/80 = -.00866$

$.2 = e^{-.008666t} \quad t = 185.8$ years

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Topics covered:

- Finding the derivatives of:
  - constants
  - powers of x, including x^1, x to fractional and negative powers
  - exponential functions with base "e" and other bases
  - products and quotients of functions
  - 6 trig functions (sin x, cos x, and tan x emphasized, but others not excluded)
  - composite functions (embedded) using the chain rule
  - equations where y is not isolated, using implicit differentiation
  - 6 inverse trig functions
  - higher order derivatives (y'', y''', etc.)
  - log functions, including natural logs and logs of other bases
  - 6 hyperbolic trig functions (sinh x, cosh x, tanh x are emphasized, but others are not excluded)
  - NO derivatives of inverse hyperbolic functions will be on the exam

- Be able to prove "a few things". You can choose one proof from a short list which will be taken from proofs in class, which include:
  - Proof of the derivatives of the constants, sum, and difference rules
  - Proof of the power rule
  - Proof of the product and quotient rules
  - Proof of the chain rule (this is too hard to put as a test question, but we did it in class)
  - Proof of $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$. These 2 proofs are more difficult than what I would put on a test, but you might be asked to prove or find a limit using these 2 facts.
  - Proof of derivative of exponent rule and derivative of log rule for bases other than e
  - Proof of derivatives of functions with sinh x, cosh x, and tanh x

- Applications and other random things to know:
  - Finding the slope/equation of a tangent line (very important)
  - Using log differentiation to simplify a product or quotient derivative
  - Using derivatives in applications where formulas are given (3.7)
  - Exponential growth/decay (3.8)
  - Related rates and rates of change (3.9)
  - Linear approximation L(x), the differential (dy), and calculating dy and Δy given a value of Δx
  - definitions of sinh x and cosh x using "e"