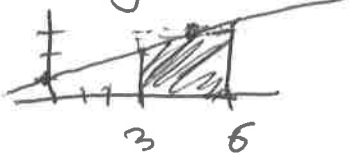
Sample Problems (Chapter 5 only) – These are the problems from an old exam – it's a small sample, not meant to be an all-inclusive list of problem types.

For problems 1 – 3, consider the function $f(x)=\frac{1}{3}x+1$ on the domain [3, 6]. Find the area under the curve using:

1. A sum of a finite number of rectangles.
2. A limit (infinite) of Riemann sums.
3. The Fundamental Theorem of Calculus (Part 2).
4. Evaluate the integrals
   a) $\int\left(\frac{x}{2}-\frac{2}{x}\right)dx$
   b) $\int_1^2\left(\frac{x}{2}-\frac{2}{x}\right)dx$
5. Evaluate the integrals
   a) $\int x\sqrt{x-1}\,dx$
   b) $\int_1^2 x\sqrt{x-1}\,dx$

1. Using $N=1$ rectangle ☺



$\Delta x = 6-3=3$
mid pt: $x=4.5$
$f(4.5)=\frac{1}{3}(4.5)+1=2.5$
Area $\approx \Delta x[f(4.5)]$
$= 3(2.5)=7.5$

2. $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x$

$\Delta x=\frac{6-3}{n}=\frac{3}{n}$

x values start at 3, increment by $\frac{3}{n}\cdot i$

So $x_i=3+\frac{3i}{n}$

Δx ↓

2. (continued)
$\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{1}{3}\left(3+\frac{3i}{n}\right)+1\right]\frac{3}{n}$

$=\lim_{n\to\infty}\frac{3}{n}\left[\sum_{i=1}^{n}2+\sum_{i=1}^{n}\frac{i}{n}\right]$

$=\lim_{n\to\infty}\left[\frac{3}{n}\cdot 2n+\frac{3}{n}\cdot\frac{1}{n}\cdot\frac{n(n+1)}{2}\right]$

$=6+\frac{3}{2}=7.5$

3. $\int_3^6\left(\frac{1}{3}x+1\right)dx=\frac{x^2}{6}+x\Big]_3^6$

$=\frac{36}{6}+6-\left(\frac{9}{6}+3\right)=7.5$

4. a) $\frac{x^2}{4}-2\ln|x|+C$

b) $\frac{3}{4}-2\ln 2=\frac{3}{4}-\ln 4$

5. a) $\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$

b) $\frac{16}{15}=1\frac{1}{15}$

Math 171 – Chapter 5 Review

- The final exam will be comprehensive, with Chapter 5 weighted more heavily, approximately 5 problems on Chapter 5, and 10 problems on the other chapters combined.
- To review for the early part of the course, see old review sheets for Exams 1 – 3, posted on my website.

Topics covered (Chapter 5 only):

- Approximating areas of graphs, equations, and tables using left and right endpoints, and midpoints
- Finding upper and lower sums
- Finding distances given velocity data or velocities given acceleration data
- Integrability – what are the conditions?
- Definition of definite integral – understanding epsilon and N
- Converting a limit of Riemann sums to an integral (and vice versa)
- Finding an integral (of a polynomial) by calculating a limit of Riemann sums
- 8 properties of integrals – how to use them to evaluate or estimate an integral
- Areas above and below the x axis (signed areas)
- The Fundamental Theorem of Calculus, Part 1
    - Finding the derivative of an integral requiring the chain rule
    - Finding an indefinite integral using the associated antiderivative
- The Fundamental Theorem of Calculus, Part 2 (Connecting an antiderivative and a definite integral)
    - Finding a definite integral using the associated antiderivative
    - Evaluating at the upper and lower limits
- Finding an indefinite integral, and evaluating a definite integral at the upper and lower limits for the following functions
    - Power functions
    - Power function with exponent = -1
    - Trig functions
    - Inverse trig functions
    - Exponential functions
    - Log functions
- Net Change Theorem – Relationship of the derivative function to the change between 2 values
- For a graph or table of values, use upper and lower estimates or the Midpoint rule to estimate changing quantities
- Use "u substitution" to find the integral of "slightly unfamiliar" functions