

Key

For the function  $f(x) = (6-x)\sqrt[3]{x}$  on the interval  $[-1, 8]$  where  $f(x)=0$  or  $f'(x)$  DNE

a) Find the critical values

$$f'(x) = (6-x)^{1/3} \cdot x^{-2/3} + x^{1/3}(-1)$$

$$= \frac{6-x}{3x^{2/3}} - \frac{x^{1/3}}{1} \cdot \frac{3x^{2/3}}{3x^{2/3}} = \frac{6-x-3x}{3x^{2/3}} = \frac{6-4x}{3x^{2/3}} = \frac{2(3-2x)}{3x^{2/3}}$$

$$f'(x) = 0 \text{ when } 3-2x=0, \quad \boxed{x = 3/2}$$

$$f'(x) \text{ DNE when } 3x^{2/3}=0, \quad \boxed{x=0}$$

b) Find any absolute and local maxima and minima on the interval, if they exist

End pts:

$$f(-1) = (6-(-1))\sqrt[3]{-1} = 7(-1) = -7$$

$$f(8) = (6-8)\sqrt[3]{8} = -2(2) = -4$$

$$f(3/2) = (6-3/2)\sqrt[3]{3/2} \approx (4.5)(1.14) \approx 5.15$$

$$f(0) = (6-0)\sqrt[3]{0} = 0$$

$$A_{\max} = L_{\max} = 4\frac{1}{2} \cdot \sqrt[3]{3/2} \approx 5.15$$

$$A_{\min} = -7$$

No  $L_{\min}$

Scoring

Baseline 4 pts

$f'(x)$ -derivative  
function 2 pts,

Crit values 2 pts,

End pt values 2 pts,

Abs max/min 2 pts

Local max/min 2 pts,

Key

For  $f(x) = x^3 - 6x^2 + 9x - 4$ 

1) Find the coordinates of any local maxima and minima

$$\begin{aligned} f' &= 3x^2 - 12x + 9 \\ &= 3(x^2 - 4x + 3) \\ &= 3(x-3)(x-1) = 0 \end{aligned}$$

when  $x = 3, 1$   
(crit. pts)

$$\begin{aligned} f(3) &= 3^3 - 6 \cdot 3^2 + 9 \cdot 3 - 4 \\ &= 27 - 54 + 27 - 4 = -4 \end{aligned}$$

$(3, -4) \leftarrow L_{\min}$

$$\begin{aligned} f(1) &= 1 - 6 + 9 - 4 = 0 \\ (1, 0) &\leftarrow L_{\max} \end{aligned}$$

2) Find the coordinates of any inflection points

$$\begin{aligned} f'' &= 6x - 12 \\ &= 6(x-2) = 0 \end{aligned}$$

when  $x = 2$

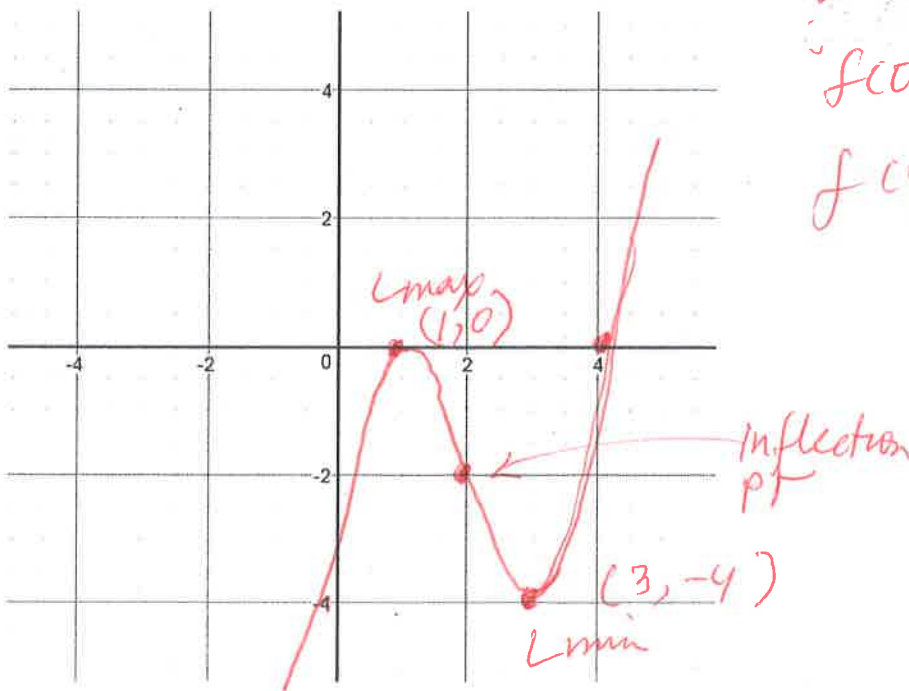
$$\begin{aligned} f(2) &= 2^3 - 6 \cdot 2^2 + 9(2) - 4 \\ &= 8 - 24 + 18 - 4 = -2 \end{aligned}$$

$(2, -2)$

Some helpful points!  
(chosen because they're  
"further out" than max/  
min)

$$f(0) = -4 \quad (0, -4)$$

$$\begin{aligned} f(4) &= 4^3 - 6 \cdot 16 + 9 \cdot 4 - 4 \\ &= 64 - 96 + 36 - 4 \\ &= 0 \quad (4, 0) \end{aligned}$$

3) Sketch the graph of the original function,  $f(x)$ 

4) List the interval(s) where the function is increasing

$(-\infty, 1)$  and  $(3, \infty)$