

1. For $f(x) = e^x(3x^2 + 5x - 4)$, find $f'(x)$

use product rule $g' \cdot f + f' \cdot g$

$$f'(x) = (3x^2 + 5x - 4) \cdot \frac{d}{dx}(e^x) + e^x(6x + 5)$$

$$= e^x(3x^2 + 5x - 4 + 6x + 5)$$

$$= e^x(3x^2 + 11x + 1)$$

2. Prove the sum rule: $\frac{d[f(x) + g(x)]}{dx} = \frac{d[f(x)]}{dx} + \frac{d[g(x)]}{dx}$

$$\lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

$$\text{or } \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

1) Find dy/dx for the curve $x^2 + 2xy + 4y^2 = 12$

By implicit differentiation

$$2x + \frac{d}{dx}(2xy) + 8y \cdot \frac{dy}{dx} = 0$$

$$2x + (2x \cdot y' + y(2)) + 8y \cdot y' = 0$$

$$\frac{y'(2x+8y)}{(2x+8y)} = \frac{-2x-2y}{2x+8y} = \frac{2(-x-y)}{2(x+4y)}$$

$$y' = \frac{-(x+y)}{x+4y} = \frac{-x-y}{x+4y}$$

← ↑
either ok

2) Find the equation of the tangent line to the curve above at the point (2, 1)

$$y' = \frac{-(2+1)}{2+4(1)} = \frac{-3}{6} = -\frac{1}{2} = \text{slope}$$

$$y-1 = -\frac{1}{2}(x-2) = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 2$$

Bonus: Find the equation of the normal line to the curve, also at (2, 1)

Normal line is perpendicular to tangent

Slope is opposite reciprocal of tangent slope

$$m = 2, \quad y-1 = 2(x-2)$$

$$y = 2x - 3$$