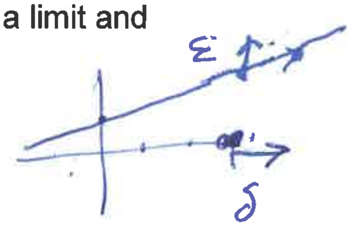
Math 181 – Week 3 Activity Name Key

1. (HW problem 2.4 # 15) Prove the statement using the ε, δ definition of a limit and illustrate with a diagram.

$$\lim_{x \to 3} \left(1 + \tfrac{1}{3}x\right) = 2$$



The x-"wiggle" is 3 times the y-"wiggle"

① choose $\delta > 0$

$|x-3| < \delta \to |1+\tfrac{1}{3}x-2| < \varepsilon$

$|x-3| < 3\varepsilon$

$\boxed{\text{Let } \delta = 3\varepsilon}$

② Show it works.

$|x-3| < 3\varepsilon$

$\left|\frac{x}{3}-1\right| < \varepsilon \to |1+\tfrac{1}{3}x-2| < \varepsilon$

so $|x-3| < \delta \to |1+\tfrac{1}{3}x-2| < \varepsilon$

2. (HW problem 2.5 # 54) Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval:

$\ln x = x - \sqrt{x}$ $(2, 3)$

Does it exist?

$f(x) = \ln x - x + \sqrt{x} \overset{?}{=} 0$

$f(x)$ is continuous on $(0, \infty)$, so it's cont. on $(2,3)$

$f(2) = \ln 2 - 2 + \sqrt{2} \approx .107$

$f(3) = \ln 3 - 3 + \sqrt{3} \approx -.169$

Since $-.169 < 0 < .107$

By the IVT, there is an $x = c$ such that $f(c) = 0$, and $2 < c < 3$

This "c" is a zero of $f(x)$

$\therefore$ the equation $\ln x = x - \sqrt{x}$ has a root