# Math 90 – Old Final Exam – key



1. Solve: $|2x - 5| < 3$.

① $2x-5<3 \Rightarrow 2x<8,\ x<4$

② $2x-5>-3 \Rightarrow 2x>2,\ x>1$

flip ↑ opp.

Graph:


solve: $(1, 4)$

2. Find the bad points (undefined points) of the expression: $\dfrac{x+1}{x^3 - 7x^2 + 10x}$.

zero in denom

$x(x-2)(x-5)$

$x \neq 0, 2, 5$

3. Simplify: $\dfrac{1+\frac{3}{2x}}{\frac{5}{x^2}}$

① Build to LCD ② Flip 2nd fraction

$$\frac{\frac{2x+3}{2x}}{\frac{5}{x^2}} = \frac{2x+3}{2x} \cdot \frac{x^2}{5} = \frac{x(2x+3)}{10}$$

or: $\dfrac{2x^2+3x}{10}$

4. Divide: $(x^4 + 7x^3 + 8x^2 - x + 2) \div (x + 2)$

By synthetic division:

$$\begin{array}{r|rrrrr} -2 & 1 & 7 & 8 & -1 & 2 \\ & & -2 & -10 & 4 & -6 \\ \hline & 1 & 5 & -2 & 3 & -4 \end{array}$$

$x^3 + 5x^2 - 2x + 3 - \dfrac{4}{x+2}$

quadratic

5. Solve for all solutions: $5q^2 - 3q + 4 = 0$

$a = 5 \quad b = -3 \quad c = 4$

$$x_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(4)}}{2 \cdot 5} = \frac{3 \pm \sqrt{-71}}{10}$$

$$= \frac{-3 \pm i\sqrt{71}}{10}$$

6. Solve for all solutions: $(2x + 5)^2 = 11$

$$2x + 5 = \pm\sqrt{11}$$

$$\frac{2x}{2} = \frac{-5 \pm \sqrt{11}}{2}$$

7. Simplify and express in standard form: $\frac{3}{2+7i} \cdot \frac{2-7i}{2-7i}$

$$= \frac{6 - 21i}{4 - 49i^2} = \frac{6}{53} - \frac{21i}{53}$$

8. Solve and graph the inequality, expressing your solution in interval notation:
$x^2 + 2x - 3 \geq 0$

Equation $x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

x-intercepts: $-3, 1$ (boundry pts)

test $(x+3)(x-1)$

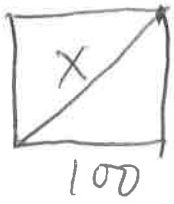
$x = -4 \quad (-)(-) \geq 0 \; +$

$x = 0 \quad (+)(-) \geq 0 \; -$

$x = 2 \quad (+)(+) \geq 0 \; +$


$(-\infty, -3] \cup [1, \infty)$

9. A square parking lot has a length of 100 feet on each side. What is the length of the diagonal connecting the northeast and southwest corners?



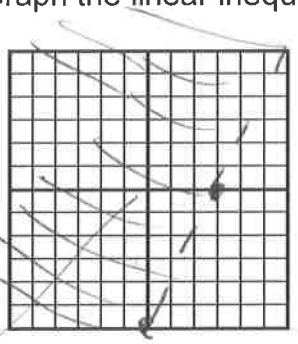

$x^2 = 100^2 + 100^2$

$x^2 = 10{,}000 + 10{,}000 = 20{,}000$

$x = \sqrt{20{,}000} = 100\sqrt{2}$ ft

$\approx 141$ ft

10. Graph the linear inequality $2x - y < 6$.



dotted

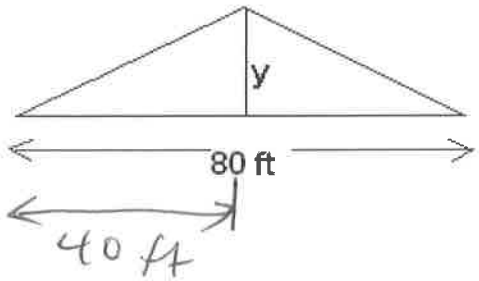
Line: $2x - y = 6$

| x | y |
|---|---|
| 0 | −6 |
| 3 | 0 |

Test (0,0): $2(0) - 0 < 6$ True

True

11. If a symmetrical roof has a 25% pitch, and the horizontal beam spanning the base of the entire roof is 80 ft. long, what is the vertical distance from the peak to the base?




$\frac{y}{40} = \frac{.25}{1}$

$y = (.25)(40) = 10$ ft

12. For the line $x - 3y = 5$:

a) Find its slope → isolate y; $-3y = -x + 5$

$y = \frac{1}{3}x - \frac{5}{3}$ $\quad m = \frac{1}{3}$

b) Find the equation of the line parallel to this line which passes through the point (6, 1).

same slope

$m = \frac{1}{3}$, thru (6, 1)

$y - 1 = \frac{1}{3}(x - 6)$ ← not final (point-slope)

$y - 1 = \frac{1}{3}x - 2$

$y = \frac{1}{3}x + 1$

slope-intercept or general form are OK final forms

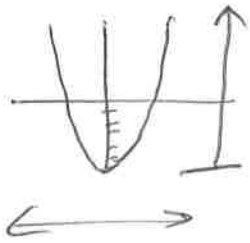
13. If $f(x) = \begin{cases} -3 & \text{for } x < 0 \\ 2x & \text{for } x \geq 0 \end{cases}$ ①, find ②

a) f(-2) = $-3$ (use 1st formula

b) f(0) = $2(0) = 0$ (use 2nd formula)

c) f(5) = $2(5) = 10$ " " "

14. Find the domain and range of $f(x) = x^2 - 7$



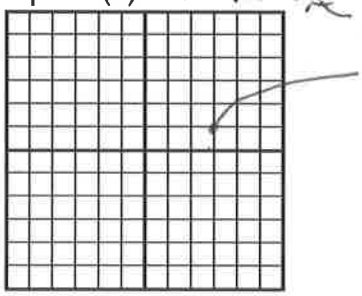
range domain: $(-\infty, \infty)$

range $[-7, \infty)$

domain

shift up 1

15. Graph: $f(x) = 1 + \sqrt{x - 3}$ shift right 3



16. If y varies inversely as t, and y = 20 when and t = 10.

a) Find the proportionality constant, k.

$y = kt$

$20 = k(10)$ ; $k = 2$

b) Write the final equation using a number for k.

$y = 10t$

17. For $f(x) = 3x - 5$, and $g(x) = x^2 + 2$, find $(f \circ g)(x)$

$f \circ g(x) = f[g(x)] = 3[x^2+2] - 5$

$= 3x^2 + 6 - 5$

$= 3x^2 + 1$

18. Find the inverse of $f(x) = \sqrt{x-3}$ for $x \geq 3$

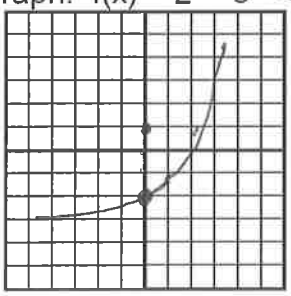
$y = \sqrt{x-3}$ switch variables

$x = \sqrt{y-3}$ isolate $y$

$x^2 = y - 3$

$y = x^2 + 3 \Rightarrow f^{-1}(x) = x^2 + 3$

19. Graph: $f(x) = 2^x - 3$



exponential

shift down

20. Solve: $\log_{10}(x) + \log_{10}(x - 3) = 1$

$\log_{10}(x \cdot (x-3)) = 1$

$\log_{10}(x^2 - 3x) = 1 \Rightarrow$

$10^1 = x^2 - 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$ ?

one log $\Rightarrow$ convert to exp. form

Check:

$x = 5$: $\log_{10} 5 + \log_{10} 2 = 1$

$\log_{10} 5 \cdot 2 = 1$ ✓

$x = -2$: $\log(-2)$

21. Use your calculator to solve for x to the nearest thousandth:

a) $x = \ln 29.6$ $= 3,388$

b) $\log x = -1.81$

$10 \quad 10$

$10^{\log x} = x = 10^{-1.81} = 0.015$

22. If the population of Riverbank is currently 20,000 and it grows exponentially at 2% annually, what will be population be in 30 years?

use e

$P(t) = 20{,}000\, e^{.02t}$

$P(30) = 20{,}000\, e^{.02(30)} = 20000\, e^{0.6}$

$= 36,442$

23. Find the value of the determinant: $\begin{vmatrix} 1 & 7 & -1 \\ -1 & -2 & 4 \\ 0 & 3 & 0 \end{vmatrix}$

$+ \quad - \quad +$

Math III doesn't cover this. It's now a Math 162 topic

$+0\begin{vmatrix} \sim & \sim \\ \sim & \sim \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -1 & 4 \end{vmatrix} + 0\begin{vmatrix} \sim & \sim \\ \sim & \sim \end{vmatrix}$

$0 - 3(4-1) = -3(3) = -9$

This page not covered in Math 111. Will be in Math 162

24. A matrix has been partially reduced as below. Solve for x, y, and z.

$$\left[\begin{array}{ccc|c} 1 & 1 & -4 & 9 \\ 0 & 1 & 5 & -7 \\ 0 & -1 & -4 & 5 \end{array}\right] \xrightarrow[r_3+r_2]{} \left[\begin{array}{ccc|c} 1 & 1 & -4 & 9 \\ 0 & 1 & 5 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\Rightarrow \boxed{z = -2}$

$y + 5(-2) = -7$

$y = -7 + 10 = 3, \boxed{y = 3}$

$x + y - 4z = 9$

$x + 3 - 4(-2) = 9$

$x + 11 = 9; \boxed{x = -2}$

$(-2, 3, -2)$

25. DO NOT SOLVE! For the word problem, "Sodas cost $1 each, burgers cost $4, and nachos cost $2. All together, 15 items are bought. If the total cost is $32, and 4 times as many sodas as nachos are bought, how many of each item is bought?"

a) Write equations to express the relationship between these quantities.

$S + B + N = 15$

$S + 4B + 2N = 32$

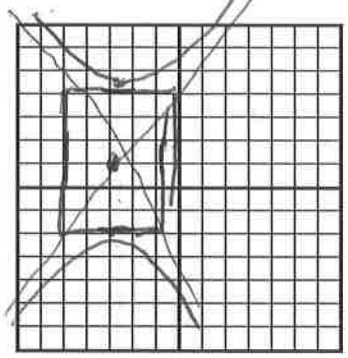
$S = 4N$

b) Write an augmented matrix to represent the system of equations.

$$\left[\begin{array}{ccc|c} S & B & N & \\ 1 & 1 & 1 & 15 \\ 1 & 4 & 2 & 32 \\ 1 & 0 & -4 & 0 \end{array}\right]$$

26. Find the center and graph of the hyperbola: $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$

Center $(-3, 1)$

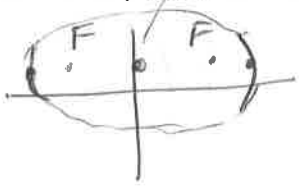


↑ vertical

$a = 2,\ b = 3$

$a = 4$ $\quad c = \sqrt{7}$

27. Find the equation of the ellipse with vertices at $(\pm 4, 1)$ and foci at $(\pm\sqrt{7}, 1)$.




c

2a 2b

Center $(0, 1)$

$c^2 = a^2 - b^2$

$7 = 16 - b^2$; $b^2 = 9$

$b = 3$

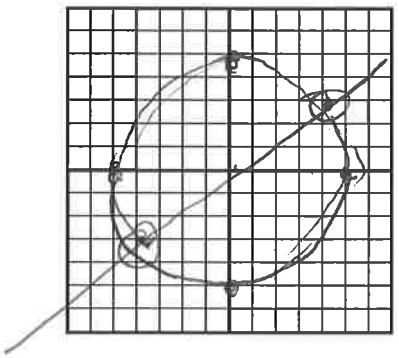
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{x^2}{16} + \frac{(y-1)^2}{9} = 1$

28. Solve the system and give coordinates of the solution point(s) You may use graphing and/or algebraic means.

$x^2 + y^2 = 25$ – circle

$3x - 4y = 0$ – line



$\frac{3x}{4} = \frac{4y}{4}$

thru. origin

$y = \frac{3}{4}x$

By substitution:

$y = \frac{3}{4}x$

$x^2 + \left(\frac{3}{4}x\right)^2 = 25$

$x^2 + \frac{9}{16}x^2 = 25$

$1\frac{9}{16}x^2 = 25$

$\frac{25}{16}$

$x^2 = 25 \cdot \frac{16}{25}$

$x^2 = 4,\ x = \pm 4$

$y = \frac{3}{4}x$

① $(4, 3)$

② $(-4, -3)$

29. Find the 60th term, $a_{60}$, of the sequence -5, -2, 1, 4, 7....

$d = -2 - (-5) = 3$
$d = 1 - -2 = 3$
common difference, arithmetic

$a_n = a_1 + (n-1)d$

$a_{60} = -5 + (60-1)(3) = 172$

30. Find the sum of the infinite sequence, $S_\infty = 2 + 2/3 + 2/9 + 2/27 + ....$

geometric

$S_\infty = \frac{a_1}{1-r} = \frac{2}{1-\frac{1}{3}}$

$= \frac{2}{\frac{2}{3}} = \frac{2}{1} \cdot \frac{3}{2} = 3$

$r = \frac{\frac{2}{3}}{\frac{2}{1}} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$
$r = \frac{\frac{2}{9}}{\frac{2}{3}} = \frac{2}{9} \cdot \frac{3}{2} = \frac{1}{3}$
common ratio, geometric

31. How many ways are there to select a committee of 2 teachers and 1 administrator if there are 8 teachers and 3 administrators to choose from?

$\binom{8}{2}\binom{3}{1} = \frac{8!}{6!2!} \cdot \frac{3!}{2!1!} = \frac{8 \cdot 7 \cdot 6!}{6! \cdot 2 \cdot 1} \cdot \frac{3 \cdot 2!}{2! \cdot 1}$

$= (28)(3) = 84$

5 people

32. How many different ways can Al, Bea, Carl, Dee, and Ed be arranged in a line if Carl must be first?

[1] · [4] [3] [2] [1] = 24

↑
Carl

33. How many different 3-digit house numbers can be formed if the digits 1, 2, 3, 4, 5, and 6 are available, and there are no repeat digits?

$\boxed{6}\ \boxed{5}\ \boxed{4} = 120$

34. Expand the polynomial $(x + 3y)^5$ $K = 5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$1x^5 + 5x^4(3y) + 10x^3(3y)^2 + 10x^2(3y)^3 + 5x(3y)^4 + 1(3y)^5$

$x^5 + 15x^4y + 10 \cdot 9x^3y^2 + 10 \cdot 27x^2y^3 + 5 \cdot 81xy^4 + 243y^5$

$= x^5 + 15x^4y + 90x^3y^2 + 270x^2y^3 + 405xy^4 + 243y^5$

35. Solve $x^3 - 8 = 0$ ← diff. of cubes

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$ $A = x$, $B = 2$

$= (x-2)(x^2 + x \cdot 2 + 2^2)$

$= (x-2)(x^2+2x+4) = 0 \Rightarrow x = 2$

$a=1\ b=2\ c=4$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)4}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2}$

3 solutions: $\{2, -1+i\sqrt{3}, -1-i\sqrt{3}\}$ $x = -1 \pm i\sqrt{3}$