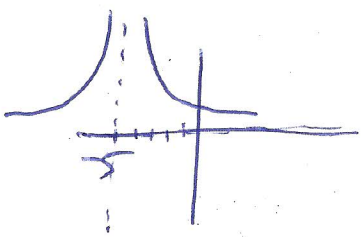
Math 111/33 Rational Functions & Inequalities Name

Rational Functions - Identify holes, vertical asymptotes, and horizontal asymptote. Then sketch the graph.

1. $f(x) = \frac{1}{(x+5)^2}$

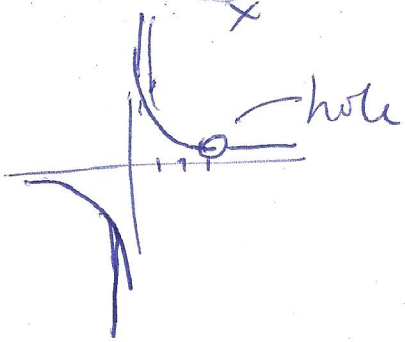
shift $\frac{1}{x^2}$ →



No holes
VA: $x=-5$
HA: $y=0$

2. $f(x) = \frac{2x-6}{x^2-3x}$ $= \frac{2(x-3)}{x(x-3)}$

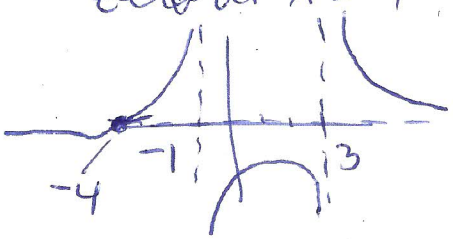
$f(x) = \frac{2}{x}$ tally ← hole at 3



3. $f(x) = \frac{(x+4)}{x^2-2x-3}$ → $\frac{x}{x^2}$ → $\frac{1}{x}$

$(x-3)(x+1)$

no holes
VA: $x=3$, $x=-1$
HA: $y=0$ (denom degree > num. degree)
zero at $x=4$



Inequalities – Graph and solve, writing your final answer in interval notation.

1. $x^3 > x$

$x^3 - x > 0$
$x(x^2-1) > 0$
$x(x+1)(x-1) > 0$

$x(x-1)(x+1)$
Test
$x=-2$ (−)(−)(−) > 0 F
$x=-\frac{1}{2}$ (+)(−)(−) > 0 T
$x=\frac{1}{2}$ (+)(−)(+) > 0 F
$x=2$ (+)(+)(+) > 0 T

$(-1,0) \cup (1,\infty)$

2. $\frac{x+5}{-x+2} > 0$ $= \frac{x+5}{-(x-2)} > 0$

$\frac{x+5}{x-2} < 0$

$(-5,2)$

Test $\frac{x+5}{x-2} < 0$

-6	$(-)/(-) < 0$	F
0	$(+)/(-) < 0$	T
3	$(+)/(+) < 0$	F

3. $\frac{3}{x-4} \le -1$

$\frac{3}{x-4} + 1 \le 0$; $\frac{3}{x-4} + \frac{x-4}{x-4} \le 0$

$\frac{3x-1}{x-4} \le 0$, $\frac{3(x-3)}{x-4} \le 0$

bad point ← $\frac{3x-1}{x-4}$; bad → $\frac{3(x-3)}{x-4}$

Test $\frac{3(x-3)}{x-4} \le 0$

0	$(-)/(-)$	F
3.5		T
4		F

$[3,4)$