

# Math III - Review of segment 3 - example problem solutions

1. *Start at bottom*

$$\begin{bmatrix} 1 & 5 & -3 & | & 4 \\ 0 & 1 & 2 & | & -5 \\ 0 & 0 & 1 & | & -3 \end{bmatrix} \begin{array}{l} \textcircled{3} \rightarrow x+5y-3z=4; x=4-5(1)+3(3)=4-5+9=-10 \\ \textcircled{2} \rightarrow y+2z=-5, y=-5-2(-3)=-5+6=1; \textcircled{1} \\ \textcircled{1} \rightarrow z=-3 \end{array}$$

$(-10, 1, -3)$

2.

$$\begin{bmatrix} 1 & 4 & 2 & | & 0 \\ 0 & 1 & -2 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{array}{l} \textcircled{3} \rightarrow x+4y+2z=0; x=-2z-4(1+2z)=-10z-4 \\ \textcircled{2} \rightarrow y-2z=1; y=1+2z \\ \textcircled{1} \rightarrow \text{inf. many sol., } z=z \end{array}$$

$(-10z-4, 1+2z, z)$

3.  $A = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 5 & 0 \\ -2 & 1 & -1 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 3 & -1 \\ 0 & 1 & 7 \\ 4 & 1 & 0 \end{bmatrix}$ ;  $2B = \begin{bmatrix} 6 & 6 & -2 \\ 0 & 2 & 14 \\ 8 & 2 & 0 \end{bmatrix}$

a)  $A+2B = \begin{bmatrix} 2+6 & 0+6 & 4+(-2) \\ 3+0 & 5+2 & 0+14 \\ -2+8 & 1+2 & -1+0 \end{bmatrix} = \begin{bmatrix} 8 & 6 & 2 \\ 3 & 7 & 14 \\ 6 & 3 & -1 \end{bmatrix}$

b)  $AB = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 5 & 0 \\ -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 3 & -1 \\ 0 & 1 & 7 \\ 4 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6+0+16 & 6+0+4 & -2+0+0 \\ 9+0+0 & 9+5+0 & -3+35+0 \\ -6+0-4 & -6+1-1 & 2+7+0 \end{bmatrix}$

$$= \begin{bmatrix} 22 & 10 & -2 \\ 9 & 14 & 32 \\ -10 & -6 & 9 \end{bmatrix}$$

4.  $\begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 4 & 3 & -2 \end{vmatrix} = i \begin{vmatrix} 0 & -1 \\ 3 & -2 \end{vmatrix} + j \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} + k \begin{vmatrix} 2 & 0 \\ 4 & 3 \end{vmatrix}$

$$= i(0 - (-3)) - j(-4 - (-4)) + k(6 - 0)$$

$$= 3i - 0j + 6k = 3i + 6k$$

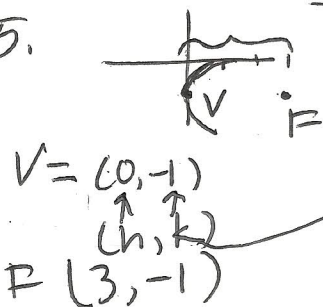
5.  $p=3$  (focal distance)

Horizontal

$$(y-k)^2 = 4p(x-h)$$

$$(y+1)^2 = 4(3)(x-0)$$

$$(y+1)^2 = 12x$$



$$6. \frac{(y+3)^2}{16} + \frac{(x-1)^2}{9} = 1$$

$$b=4$$

$$a=3$$

$$\text{Center } (h, k) = (1, -3)$$

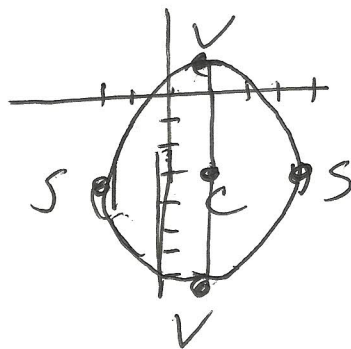
$$\text{Vertices } (1, -3 \pm 4)$$

$$\text{semi-vert: } (1 \pm 3, -3)$$

$$\text{Foci: } c^2 = 4^2 - 3^2 = 16 - 9 = 7$$

$$c = \sqrt{7}$$

$$(1, -3 \pm \sqrt{7})$$



$$7. x^2 - 9y^2 = 36$$

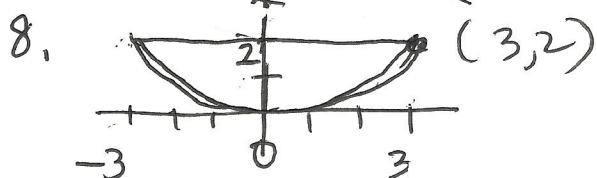
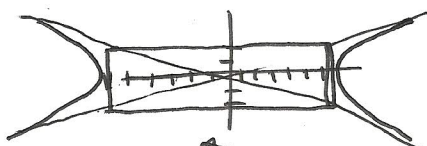
get 1 on one side  $\Rightarrow$  divide by 36

$$\frac{x^2}{36} - \frac{y^2}{4} = 1$$

$$a=6 \quad b=2$$

Center (0,0)

$x^2$  pos  $\rightarrow$  horizontal hyperbola



Choose vertex at origin (0,0)

$$d = 6 \text{ in, } r = 3 \text{ in}$$

Vertical parabola:  $x^2 = 4py$

(3,2) is on parabola

$$3^2 = 4p(2) \quad ; \quad p = \frac{9}{8} = 1\frac{1}{8}$$

= focal distance

The light bulb is at focus, (0,  $1\frac{1}{8}$ ) or

$1\frac{1}{8}$  in. above center of mirror

$$9. A = \frac{1}{2} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{vmatrix} \leftarrow \text{abs. value of det} = \frac{1}{2} \begin{vmatrix} 2 & 0 & 7 \\ -5 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix} \left. \begin{array}{l} \text{expand along} \\ \text{3rd row} \end{array} \right\}$$

$$= \frac{1}{2} \left[ +1 \begin{vmatrix} 0 & 7 \\ 3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 2 & 7 \\ -5 & 4 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ -5 & 3 \end{vmatrix} \right]$$

$$= \frac{1}{2} [(0-21) - (8 - -35) + (6-0)] = \frac{1}{2} (-21 - 43 + 6)$$

$$= \frac{1}{2} (-58) = -29$$

$$A = |-29| = 29$$