Math 111 – Exam 2 Name Key

When asked to solve, give all real and imaginary solutions.

Part I: Shorter Problems (6 points each)

1. Solve for x: $2x^2 - x = 7$

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$2x^2 - x - 7 = 0$

$a=2 \quad b=-1 \quad c=-7$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$

$= \frac{1 \pm \sqrt{1+56}}{4} = \frac{1 \pm \sqrt{57}}{4}$ (fully simplified)

2. Solve for x: $x^4 - x^2 - 12 = 0$

let $u = x^2$

$u^2 - u - 12 = 0$

$(u-4)(u+3) = 0$

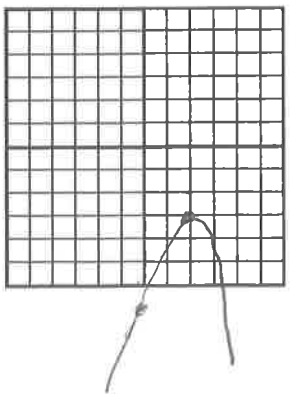
$u = 4 \Rightarrow x^2 = 4, \quad x = \pm 2$

$u = -3 \Rightarrow x^2 = -3, \quad x = \pm i\sqrt{3}$

or $\{-2, 2, -i\sqrt{3}, i\sqrt{3}\}$

3. Find the vertex and graph of $f(x) = -x^2 + 4x - 7$

face down



Vertex: $x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$

$y = f(2) = -(2^2) + 4(2) - 7$

$= -4 + 8 - 7 = -3$

$(2, -3)$

Another pt: y-int: let $x = 0$

$f(0) = -7 \quad (0, -7)$

4. Solve the absolute value equation:
$|2x - 3| = 7$

① $2x-3=7$

$2x=10$

$x=5$

② $2x-3=-7$

$2x=-4$

$x=-2$

$\{5, -2\}$

5. Solve the absolute value inequality, and express your answer in interval notation:
$|x + 2| \geq 5$

① $x+2 \geq 5$

$x \geq 3$

② $x+2 \leq -5$

$x \leq -7$

$(-\infty, -7] \cup [3, \infty)$

6. a) Find the distance between the points (6, -1) and (0, 2)

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-6)^2+(3)^2} = \sqrt{36+9} = \sqrt{45}$ or $\sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

b) Find the midpoint of the 2 points in part a)

$\bar{x} = \frac{6+0}{2} = 3, \quad \bar{y} = \frac{-1+2}{2} = \frac{1}{2}$ $(3, \frac{1}{2})$

7. Solve the inequality, and express your answer in interval notation:
$x^2 - 3x \leq 10$

$x^2-3x-10 \leq 0$

$(x-5)(x+2) \leq 0$

boundary pts: $x=5, -2$

$[-2, 5]$

Test	$(x-5)(x+2) \leq 0$
$x=-3$	$(-)(-) \leq 0$ F
$x=0$	$(-)(+) \leq 0$ T
$x=6$	$(+)(+) \leq 0$ F

8. Solve the inequality, and express your answer in interval notation:

$\frac{x-3}{x+7} \geq 0$

boundary: 3, −7

−bad pt (not included)

$(-\infty, -7) \cup (3, \infty)$

Test

$x = -8$ $\quad (-)/(-) \geq 0$ T

$x = 0$ $\quad (-)/(+) \geq 0$ F

$x = 4$ $\quad (+)/(+) \geq 0$ T

$x-3/x+7 \geq 0$

9. Write the equation of the circle in standard form, then give the center and radius.

$x^2 + y^2 - 6y = 16$

Center: (0, 3)

Radius: 5

$x^2 + y^2 - 6y + 9 = 16 + 9$

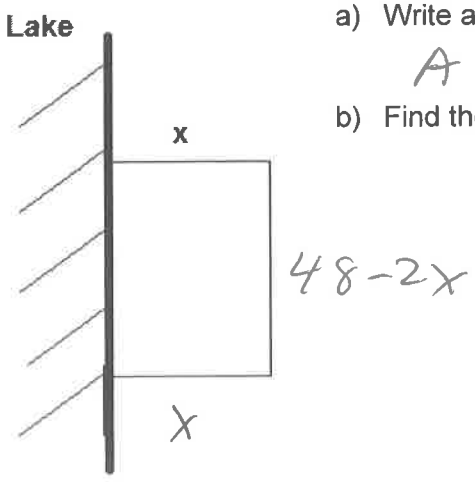
$n = 6/2 = 3$

$nn = 3^2 = 9$

$x^2 + (y-3)^2 = 25$

$(0, 3)$, $r = \sqrt{25} = 5$

10. A 48-foot rope is used to section off 3 sides of a rectangular picnic area on the edge of a lake. The lake's edge is longer than 48 ft.; there is no rope on the lake's edge.



a) Write an expression for the area, and call it A(x), where x is the width.

$A = L \cdot W = (48-2x)x = -2x^2 + 48x$

b) Find the dimensions of the rectangle that produce the maximum area.

Vertex: $x = \frac{-b}{2a} = \frac{-48}{2(-2)}$

$= 12$ ft

$W = 12$ ft

$L = 48 - 2(12) = 24$ ft

11. Use synthetic division to divide. Write your final answer as a polynomial.

$$\frac{x^4 - 7x^3 + 6x^2 - 2x + 3}{x - 2}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -7 & 6 & -2 & 3 \\ & & 2 & -10 & -8 & -20 \\ \hline & 1 & -5 & -4 & -10 & -17 \end{array}$$

$x^3 - 5x^2 - 4x - 10 - \frac{17}{x-2}$

Bonus (2 pts): If $f(x) = x^4 - 7x^3 + 3x^2 - 2x + 3$, what is $f(+2)$?

By Remainder Theorem, $f(+2) = -17$ (Should have been 0 if no typo)

OR $f(2) = 2^4 - 7\cdot 2^3 + 3\cdot 2^2 - 2\cdot 2 + 3$

$= 16 - 56 + 12 - 4 + 3$

$= -29$

Part II – Longer Problems (points as marked)

1. (10 pts) Find all real and imaginary zeros of $f(x) = x^3 + 3x^2 - 6x - 8$, and write f(x) in factored form.

1·1 1·8 2·4

Possible zeros: $\pm 1, \pm 2, \pm 4, \pm 8$ (Your guesses may differ)

Try $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -6 & -8 \\ & & 1 & 4 & -2 \\ \hline & 1 & 4 & -2 & (-10) \end{array}$$

Not a zero

Try $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -6 & -8 \\ & & 2 & 10 & 8 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

good zero

$(x-2)(x^2+5x+4)$

$f(x) = (x-2)(x+4)(x+1)$ ← factored form

zeros: $x = 2, -4, -1$

2. (12 pts) For the polynomial: $f(x) = -2x^2(x+3)^3(x-2)$

a) What is the degree of the polynomial? 6

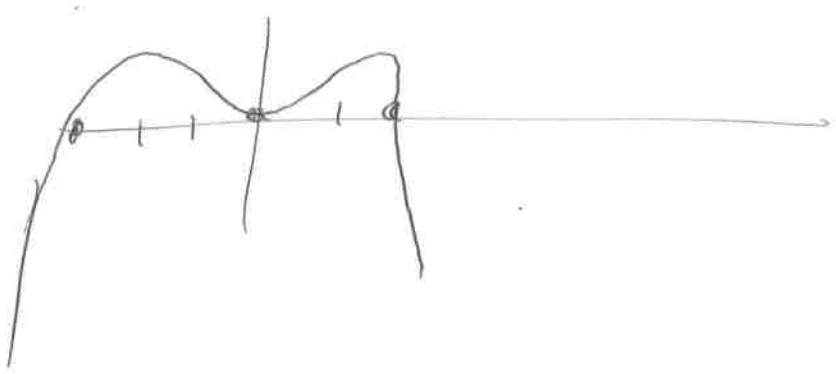
b) Circle the description that best applies to f(x):

rising falling face up (face down)

c) List each zero and its multiplicity.

Zero:	0	−3	2
Multiplicity:	2	3	1
	touch	cross	cross

d) Draw a rough sketch of the graph, with attention given to the position of the zeros, and the behavior of the graph at those points. The height of the graph is less important.



3. (12 pts) For $f(x) = \frac{2x^2}{x^2-9}$ $= \frac{2x^2}{(x-3)(x+3)}$

a) Give the value(s) of all zeros

$x = 0$

b) Give the equation(s) of all vertical asymptotes

$x = 3, -3$

c) Give the equation(s) of any horizontal asymptote(s)

highest powers $y = \frac{2x^2}{x^2} = 2$ $y = 2$

d) Draw a rough sketch of the graph