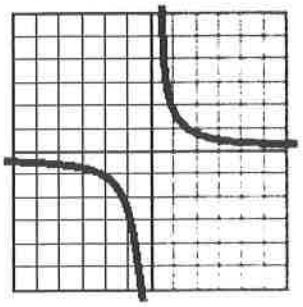
Math 111 – Exam 2 Name Key

Part I – Short Answer (2 pts. each, 14 pts. total for this part)

1. For the graph below, circle all descriptions which are true of the graph



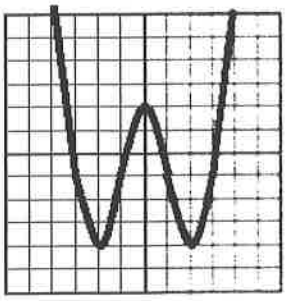
is even

is odd

is one-to-one

has asymptote(s

2. For the graph below, circle all descriptions which are true of the graph



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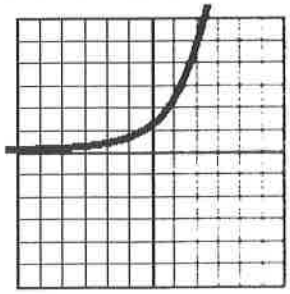
3. For the graph below, circle all descriptions which are true of the graph



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4. A polynomial has $x = 1 - 7i$ as one of its zeros. Give the value of one other zero.

$x = 1 + 7i$

5. Solve: $x = \log_2 \frac{1}{16}$

$2^x = \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

$x = -4$

6. Solve and round to the nearest hundredth: $\ln x = 0.581$

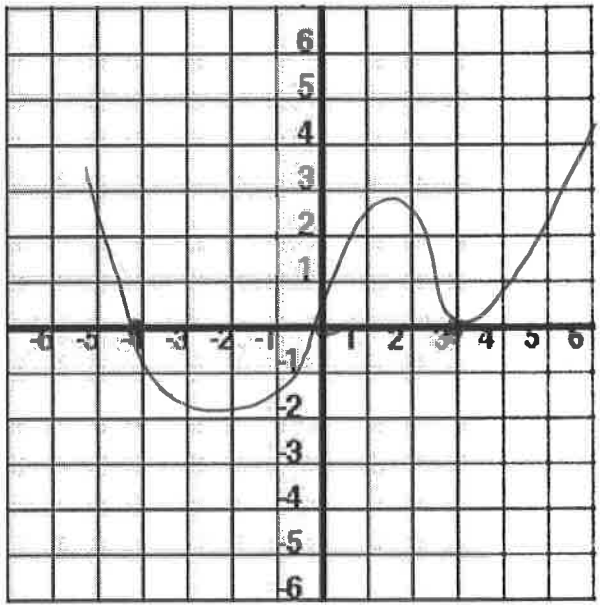
$e^{\ln x} = x = e^{.581} =$

7. Solve and round to the nearest hundredth: $x = \log 1.463$

$x \approx .17$

Part II – Short(er) Problems – 6 pts each (total of 36 pts for this part)

1. A 4th degree polynomial has zeros with multiplicities as listed. The coefficient of the highest power term ("a") is positive.



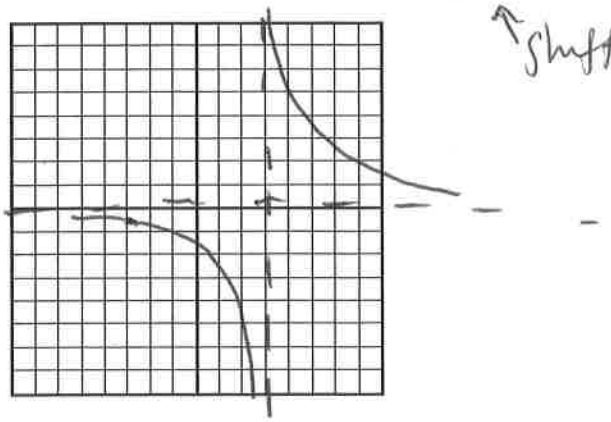
Zero:	-4	0	3
Multiplicity:	1	1	2

a) Sketch a possible graph

b) Write a possible polynomial function with the given zeros and multiplicities

$f(x) = (x+4)(x)(x-3)^2$

2. a) Graph the equation f(x) = $y = \frac{1}{x-3}$



↑ shift right

b) Give the equations of any horizontal or vertical asymptotes:

HA: $y = 0$

VA: $x = 3$

3. Let $f(x) = x^2 + 3$, for $x \geq 0$. Calculate $f^{-1}(x)$, and give the domain of $f^{-1}(x)$.

$y = x^2 + 3$

$x = y^2 + 3$

$x - 3 = y^2$

$y = \sqrt{x-3}$ (+ only)

$f^{-1}(x) = \sqrt{x-3}$ ← inside ≥ 0, $x - 3 \geq 0$

Domain $\{x \mid x \geq 3\}$

4. The polynomial $f(x) = x^3 - 9x^2 + 23x - 15$ has $k = 3$ as one of its zeros. Find the other zeros of f(x), including both real and imaginary zeros.

$$\begin{array}{r|rrr} 3 & 1 & -9 & 23 & -15 \\ & & 3 & -18 & 15 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$x^2 - 6x + 5$

$(x-5)(x-1) = 0$

Zeros: 3, +5, +1

$f(x) = (x-3)(x-5)(x-1)$

5. For an exponential function f(x), f(0) = 3 and f(2) = 12. Find the values of C and a, and write the final function in the form $f(x) = Ca^x$

$f(0) = 3 \Rightarrow (0, 3)$

$f(2) = 12 \Rightarrow (2, 12)$

plug in: $3 = Ca^0 = C$; $C = 3$

$12 = Ca^2$

$12 = 3 \cdot a^2$; $a^2 = 4$, $a = \pm 2$ pos only base

$y = f(x) = 3(2)^x$

6. A tablet has an initial value of $600 and depreciates exponentially at a rate of 40% each year.

a) Write a function, A(t), which describes the value of the tablet over time.

$A(t) = 600e^{-.4t}$

b) Calculate A(3)

$A(3) = 600e^{-.4(3)} = 600e^{-1.2} =$

c) What "real life" quantity does A(3) describe?

The value of the tablet after 3 years

Part III - Long(er) Problems (10 pts. each, total of 40 points for this part)

1. A parking attendant can wait on a maximum of 10 vehicles/minute. If vehicles leave randomly at an average rate of x vehicles/minute, the average wait time, T, in minutes is

$T(x) = -\frac{1}{x-10}$

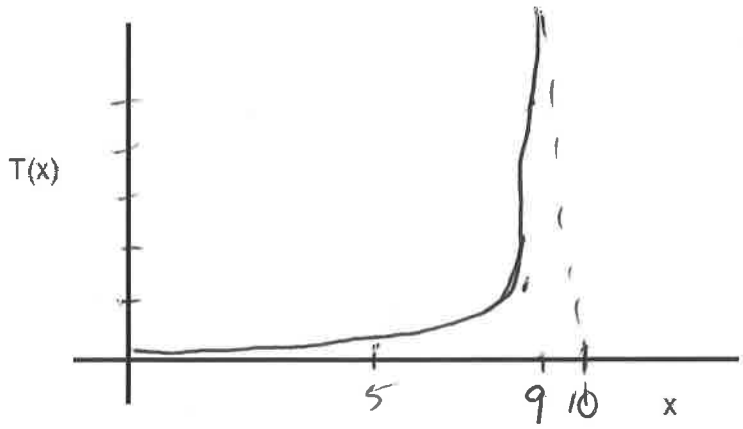
a) Calculate T(0), T(9), and T(9.8)

$T(0) = -\frac{1}{0-10} = \frac{1}{10}$

$T(9) = -\frac{1}{9-10} = +1$

$T(9.8) = -\frac{1}{9.8-10} = -\frac{1}{-.2} = 5$

b) Draw a rough sketch of the graph of this function (you may calculate more points if you need to).



c) What happens to the wait time as the average rate of vehicle arrival approaches 10 cars/minute? It increases rapidly

d) What happens to the wait time if the average rate of vehicle arrival equals or exceeds 10 cars/minute? It goes to infinity $\to \infty$

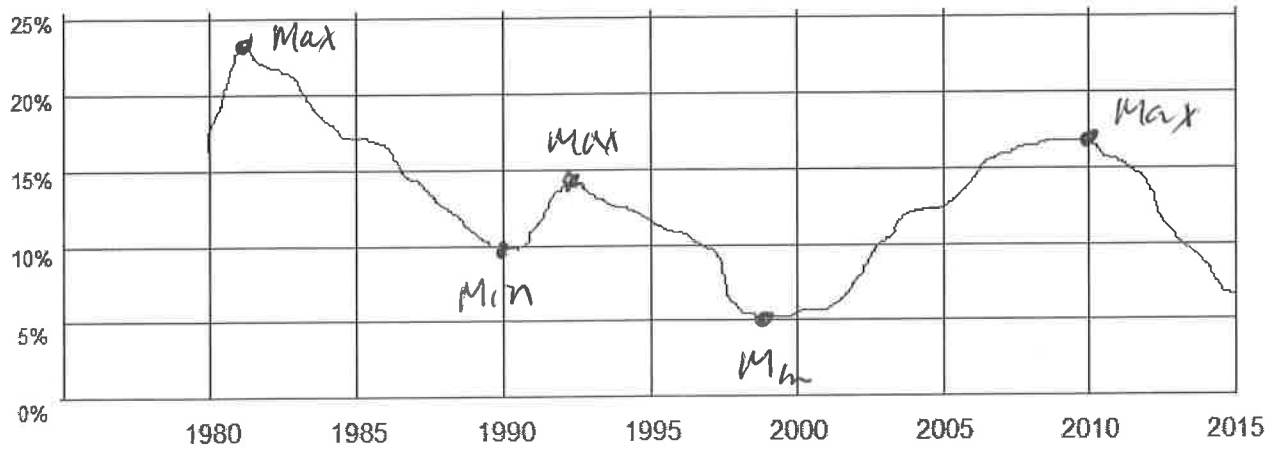
$\frac{-1}{10-10} = -\frac{1}{0}$

e) What is the "practical domain" of this function (i.e. the domain based on what happens in real life?)

$[0, 10)$ ← not including 10

2. Below is a graph of unemployment rates. Disregard "tiny wiggles" and concentrate on the major trends. Give values to the best of your ability to the nearest unit.

Unemployment Rate for Young Adults (18 – 25) by Year



a) List all local maxima

23%, 14%, 17%

b) List all local minima

10%, 5%

c) List the absolute maximum and the absolute minimum.

$A_{max} = 23\%$ $A_{min} = 5\%$

d) In approximately what year was the unemployment rate the highest? The lowest?

Highest: 1981 Lowest 1998

e) Based on where your young adult years fall/fell comment on how the graph's employment numbers affect you and your friends of similar age.

I graduated in 1983. The job market was horrible! It took a long time for many of my friends to find a job.

3. A sum of $1000 is borrowed at 18% annual interest for 20 years

a) Find the amount owed if the interest is compounded annually.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{.18}{1}\right)^{1 \cdot 20}$$

b) Find the amount owed on $1000 at 18% annual interest over 20 years if the interest is compounded continuously.

$A = Pe^{rt} = 1000e^{(.18)(20)}$

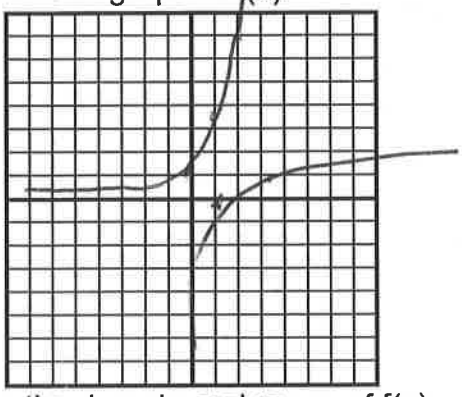
=

c) What causes the values in parts a) and b) to differ?

The base of the exponents differs for annual vs. continuous

4. Let $f(x) = e^x$

a) Sketch the graph of f(x)



b) Give the domain and range of f(x)

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

c) What is $f^{-1}(x)$, the inverse of f(x)?

Inverse of e^x is $\ln x$

$f^{-1}(x) = \ln x$

d) Graph $f^{-1}(x)$ on the same grid as the graph in part a)

e) Give the domain and range of $f^{-1}(x)$.

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

f) The graphs of f(x) and $f^{-1}(x)$ are symmetrical across what line? Give the equation.

$y = x$