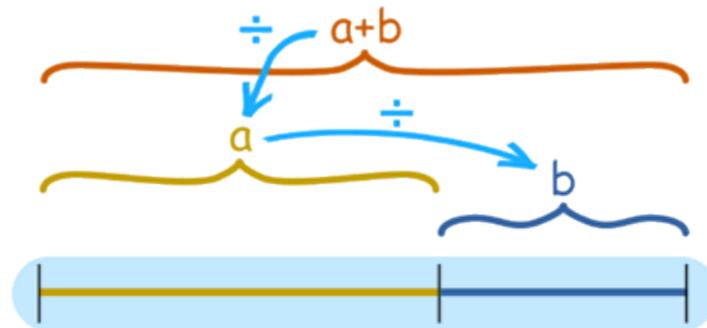# GOLDEN RATIO

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THE GOLDEN RATIO

WHAT IS THE GOLDEN RATIO?

"The golden ratio is a special number found by dividing a line into two parts so that the whole length divided by the long part is also equal to the long part divided by the short part." (MATHSISFUN.COM)

$$\frac{a+b}{a} = \frac{a}{b} = 1.618... = \varphi$$

It has many other names, Phi, Golden mean, Golden section, Divine portion, and the Divine section.

In mathematics it is most often referred to as the Greek letter Phi or represented by its symbol "Φ"

It is an irrational number with an infinite number of digits after the decimal point, but is most often it is recognized by just 1.61803398…

In an equation form, it looks like:

a/b = (a+b)/a = 1.6180339887498948420…(Livescience.com)

The Golden Ratio is often visually represented as the Golden Rectangle
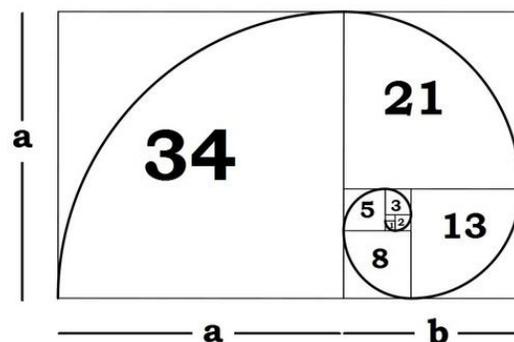
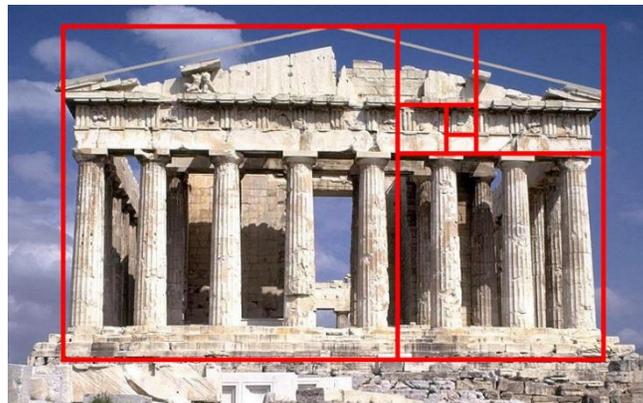It is thought to be the most appealing to the eye rectangle.

In western philosophy the Golden mean is also thought as "dividing line", or the "middle way". It is the separation, or the middle between two extremes, one of excess and the other of deficiency. (newworldencyclopedia.org)

HISTORY OF THE GOLDEN RATIO:

The golden has always existed and it is actually unknown when it was first discovered, but we know that it may have been used in Egyptian and Greek architecture. Also, it is believed to have been applied to the design of the Parthenon. The golden ratio has held people's attention for over 2,000 years (Mario Livio).

> "Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics" - Mario Livio

Many mathematicians, artist, and philosophers have used the golden ratio throughout history. One of the earliest records of it use was from Phidias. Phidias was a mathematician and the greatest sculptor of his time. He studied and applied the golden ratio to the design of the sculptures for the Parthenon. The golden ratio was also named Phi after Phidias (500 BC- 432 BC).

Another important historical figure that mentioned his use of the golden ratio in his studies was Plato (428 BC- 347 BC). In "Timaeus" Plato describes five possible regular solids that relate to the golden ratio. He also considers the golden ratio to be the most binging of all mathematic relationships.

Euclid (323 BC- 265 BC) was the first to give a recorded definition of the golden ratio as "a dividing line in the extreme and mean ratio" in his book named *Elements.* He was also the one to link the number to the construction of the pentagram.

Fibonacci (1770- 1250) discovered a sequence now known as the Fibonacci sequence which happened to approach the ratio asymptotically. The Fibonacci sequence is a "series of numbers in which each number (*Fibonacci number*) is the sum of the two preceding numbers. The simplest is the series 1, 1, 2, 3, 5, 8, etc." (Fibonacci sequence)

Most commonly the Golden Ratio is allied to Leonardo da Vinci for his use of it in his painting of *The Last Supper.* In this painting Leonardo da Vinci used the golden ratio to define the fundamental portions of the painting, rationalizing the body proportions as well as the wall proportions. In 1509, Luca Pacioli published a dissertation titled *De Divina Proportione* which had illustrations created by Leonardo da Vinci. It was probably da Vinci who first called it the

"section aurea" which is Latin for Golden Section. Many renaissance artist used the golden mean in paintings and sculptures to achieve balance.
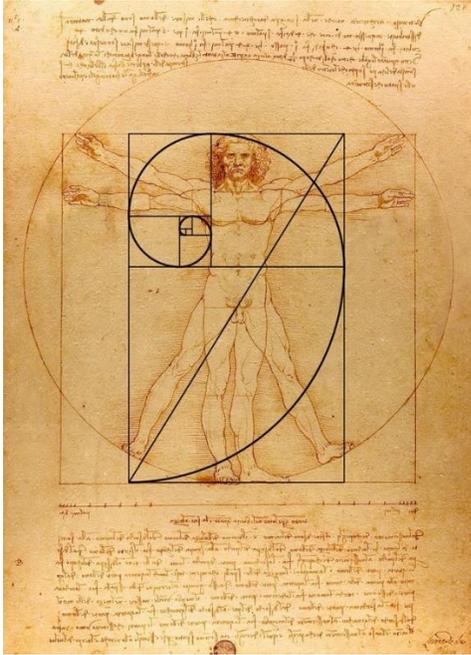
The *Vitruvian Man* by Leonardo da Vinci and a Golden Mean drawing.
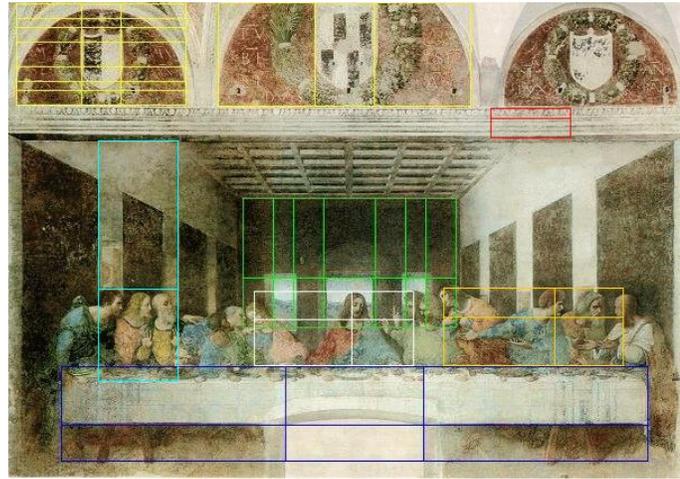
The Last Supper by Leonardo da Vinci and a Golden Mean drawing.

The term Phi was not used until the 1900's when Mark Barr an American mathematician used the Greek letter Φ, after the late mathematician and sculptor Phidias.

More recently, the 1970's, Phi was used to help discover a pattern now known as Penrose Tiles. A **Penrose tiling** is a non-periodic **tiling** generated by an aperiodic set of prototiles (Penrose Tiles).

Today the Golden ratio is usually a concept that is applied in art, design and architecture.

CALCULATION OF THE GOLDEN RATIO:

One of the easiest ways to derive the Golden Ratio is by:
Dividing 1 by a number (for example the number 2): ½=0.5
Then adding 1: 0.5+1=1.5
Giving you a new number, then you repeat dividing 1 by the new number till you get to the Golden Ratio. (MATHSISFUN.COM)
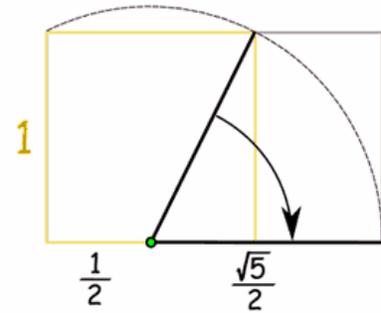
Another way of deriving the Golden Ratio is by drawing it:

To draw it you start by drawing a 1" square.
Then place a dot half way along one side
and draw a line from the point to the opposite corner (the
length will be $\sqrt{5}/2$)
then turn the line so that it runs along the side of the square.

You can also use the measurements to create a formula for Phi:

$$\phi = \frac{1}{2} + \frac{\sqrt{5}}{2} = \frac{1 + \sqrt{5})}{2}$$

The square root of 5 is ≈ 2.236068
Therefore the Golden Ratio is $\approx \frac{1+2.236068}{2} = \frac{3.236068}{2} = 1.618034..$

(MATHSISFUN.COM)

Like Pi (π=3.14159..), Phi is also an irrational number, but Phi is the solution to a quadratic equation.

Deriving the value of the Golden Ratio from the Quadratic Equation:

A=1, b=-1, and c=-1

$$x = \frac{-b \pm \sqrt{b^2} - 4ac}{2a} = \frac{-(-1) \pm \sqrt{-1^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

Then again, the square root of five is ≈ 2.236068

Therefore the Golden Ratio is $\approx \frac{1+2.236068}{2} = \frac{3.236068}{2} = 1.618034 \ldots$

The Golden Ratio is also a number that is equal to its own reciprocal plus one:
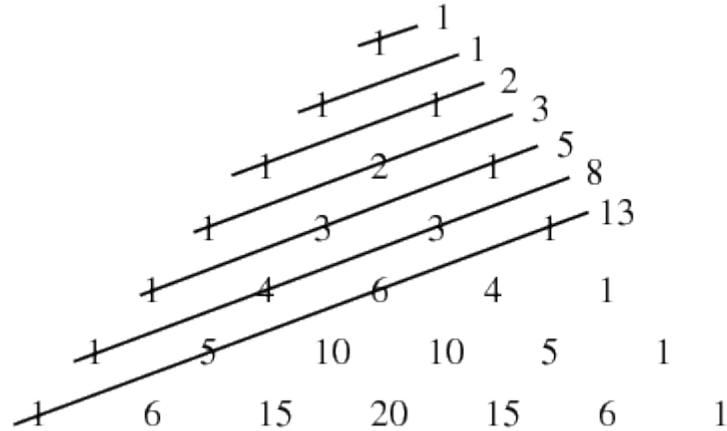
$$\phi = \frac{1}{\phi} + 1$$

Phi can also be derived using the Fibonacci sequence. The Fibonacci sequence begins with 0 and 1, and each new number in the sequence is the sum of the two before it.
Example of the sequence:
1 + 1 = 2   2 + 3 = 5
1 + 2 = 3   3 + 5 = 8

The same pattern can be is created by using Pascal's Triangle:

The Golden Ratio's relationship to the Fibonacci sequence can be found dividing each number *by the one before it, like*:

$$1/1 = 1$$
$$2/1 = 2$$
$$3/2 = 1.5$$
$$5/3 = 1.666$$
$$13/8 = 1.625$$
$$21/13 = 1.615$$

The further you go in the series the closer you get to the Golden Ratio. For example: 233 /144 = 1.61805 …

GEOMETRY AND THE GOLDEN RATIO:

The Golden Ratio is the classic shape of Phi. Phi appears alongside the perfect (unit) square:

The Golden Rectangle has dimensions 1\times\ phi such that removing the unit square is left with the rectangle $(\phi - 1) \times 1$ similar to the original rectangle the most fundamental property of the golden ratio is:

1. $\phi : 1 = 1 : (\phi - 1)$

To see this, we reduce 1 to

2. $\phi^2 - \phi - 1 = 0$

With two solutions $(1 \pm \sqrt{5})/2$ one being positive or negative.
The negative solution would actually be the conjugate of Phi ϕ'.

(CUT-THE-KNOT.ORG)

The Golden Ratio in geometry it appears in basic construction of an equilateral triangle, square and pentagon placed inside of a circle, and in more complex three-dimensional solids.
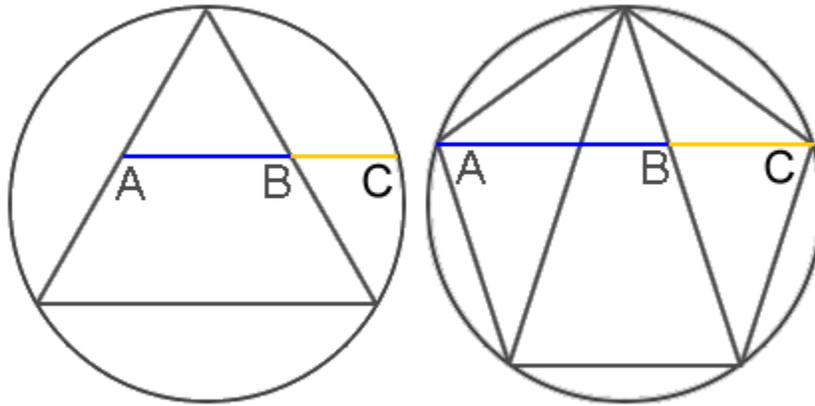

(GOLDENNUMBER.NET)

APPLING THE GOLDEN RATIO TO ART:

The Golden Ratio can be used to achieve beauty, balance and harmony in art, architecture and design. It can be used as a tool in art and design to achieve balance in the composition.

The best examples of use of the Golden Ratio in art are by renaissance artist Leonardo da Vinci, Michelangelo, and Raphael.

Leonardo di ser Piero da Vinci was a man of many interest. Those interest being invention, painting, sculpting, architecture, science, music, mathematics, engineering, literature, anatomy, geology, astronomy, botany, writing, history, and cartography. Leonardo used the Golden section in many of his paintings, and drawings. During the Renaissance it was known as The Divine Proportion. In his famous painting *The Last Supper,* all of the proportions are balanced because he used the Golden Portion.
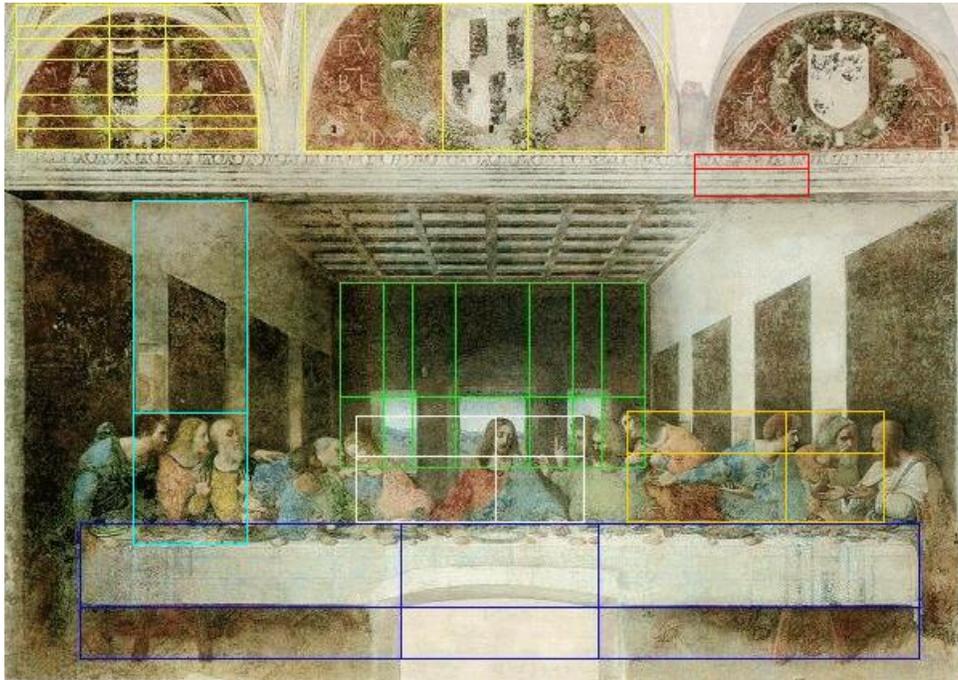

Michelangelo di Lodovico Buonarroti Simon, was a painter, architect, poet, and engineer from the Renaissance. He was considered the greatest living artist of his time. Most commonly Michelangelo is remembered by his painting *The Creation of Adam* that is on the selling of the *Sistine Chapel.* It is in this painting that one can see how Michelangelo used the Golden ratio. God's finger and Adam's finger meet at the Golden ratio point of the area that contains them.
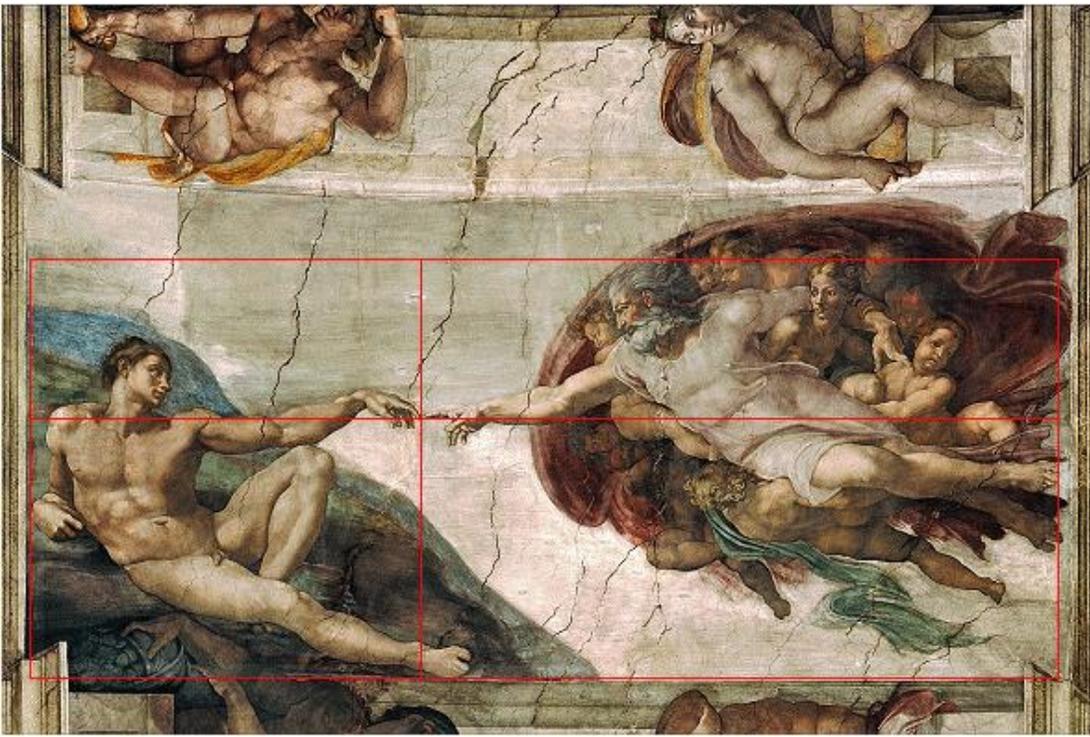
Raffaello Sanzio da Urbino, was also a painter and architect from the Renaissance. In his painting *The School of Athens* the division between the figures in the painting and their proportions are distributed using the Golden ratio.
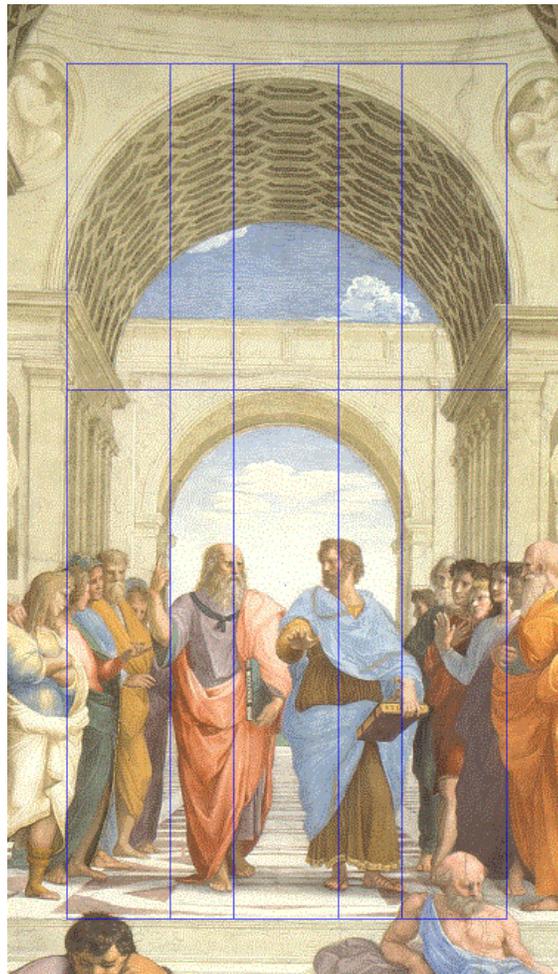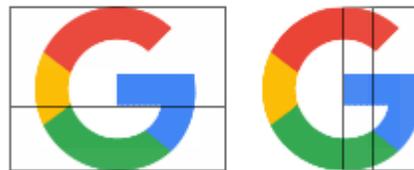
The best example of the golden ratio being used today is in logos. The design of logos usually consist of creating an appealing, well balanced and memorable symbol. Google's logo is one of the best examples of the use of the Golden ratio.
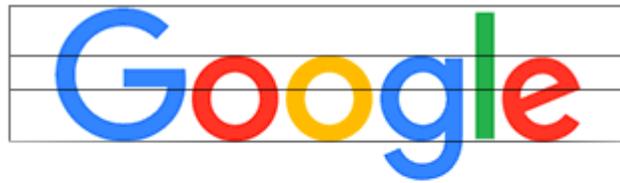
Top to bottom of lower case letters

Top to bottom of lower case g

www.goldennumber.net

Throughout their logo we can see how they evenly distribute the letters and colors using the golden ratio. (GOLDENNUMBER.NET)

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